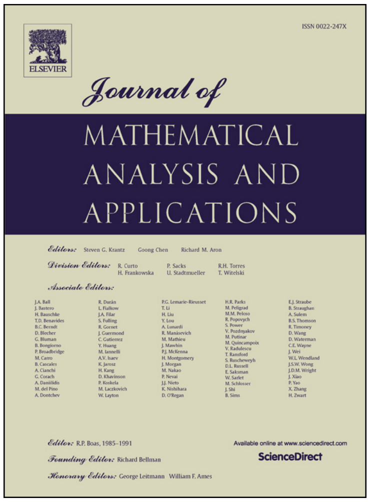

Asymptotic spectrum of the linear Boltzmann equation with general boundary conditions in finite bodies

Youssouf Kosad, Khalid Latrach

PII: S0022-247X(16)30672-2
DOI: http://dx.doi.org/10.1016/j.jmaa.2016.10.067
Reference: YJMAA 20842

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 12 July 2016

Please cite this article in press as: Y. Kosad, K. Latrach, Asymptotic spectrum of the linear Boltzmann equation with general boundary conditions in finite bodies, *J. Math. Anal. Appl.* (2017), http://dx.doi.org/10.1016/j.jmaa.2016.10.067

Asymptotic spectrum of the linear Boltzmann equation with general boundary conditions in finite bodies

Youssouf Kosad[a,1], Khalid Latrach[a,*]

[a] *Université Blaise Pascal (Clermont II), Laboratoire de Mathématiques, CNRS UMR 6620,*
Campus Universitaire des Cézeaux, 3 place Vasarely, TSA 60026 CS 60026,
63178 Aubière, France

Abstract

The purpose of this paper is the study of the spectral properties of both streaming operator and transport operator with general boundary conditions in multidimensional bounded geometry. We discuss the asymptotic spectrum: existence and nonexistence results of eigenvalues in the half-plane $\{\lambda \in \mathbb{C} : \mathrm{Re}\lambda > s(T_H)\}$ where $s(T_H)$ stands for the spectral bound of the streaming operator T_H. Next, we discuss the irreducibility of the transport semigroup. In particular, we establish that the transport semigroup is irreducible if the boundary operator is strictly positive. Afterwards, we discuss the strict monotonicity of the leading eigenvalue (when it exists) of the transport operator with respect to different parameters of the equation. Our analysis is based essentially on results from the theory of positive linear operators.

Keywords: Transport operator, general boundary conditions, asymptotic spectrum, positivity in the lattice sense, irreducibility, leading eigenvalue.
2010 MSC: 47A10, 47A55, 35Q20

---

1. Introduction

This paper is devoted to spectral properties of transport equation in finite bodies, when the behavior at the boundary is governed by a positive boundary operator H relating the incoming flux to the outgoing one. More precisely, we are concerned with the description of the asymptotic spectrum of the integro-differential operator

$$\begin{aligned} A_H\psi(x,v) &= -v\cdot\nabla_x\psi(x,v) - \sigma(x,v)\psi(x,v) + \int_{\mathsf{V}} \kappa(x,v,v')\psi(x,v')d\mu(v') \\ &= T_H\psi + K\psi \end{aligned}$$

where $(x,\, v) \in \mathsf{D}\times\mathsf{V}$. Here D is a smooth open subset of $\mathbb{R}^n$, $\mu(\cdot)$ is a positive Radon measure on $\mathbb{R}^n$ such that $\mu(0) = 0$ with support V. The functions $\sigma(\cdot,\cdot)$ and $\kappa(\cdot,\cdot,\cdot)$ are called, respectively, the collision

---

[*] Corresponding author
Email addresses: `Kosadabdi@math.univ-bpclermont.fr` (Youssouf Kosad), `Khalid.Latrach@math.univ-bpclermont.fr` (Youssouf Kosad), `Khalid Latrach@math.univ-bpclermont.fr` (Khalid Latrach)

Preprint submitted to Journal of LaTeX Templates *October 27, 2016*