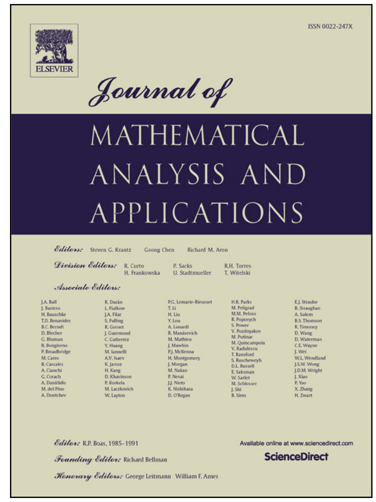
Accepted Manuscript



Several Constructions in the Eremenko-Lyubich Class

Kirill Lazebnik

PII: S0022-247X(16)30690-4
DOI: http://dx.doi.org/10.1016/j.jmaa.2016.11.007
Reference: YJMAA 20860

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 6 April 2016

SEVERAL CONSTRUCTIONS IN THE EREMENKO-LYUBICH CLASS

KIRILL LAZEBNIK

Abstract. We use a theorem of Bishop in [Bis15] to construct several functions in the Eremenko-Lyubich class $\mathcal{B}$. First it is verified, that in Bishop's initial construction [Bis15] of a wandering domain in $\mathcal{B}$, all wandering Fatou components must be bounded. Next we modify this construction to produce a function in $\mathcal{B}$ with wandering domain and uncountable singular set. Finally we construct a function in $\mathcal{B}$ with unbounded wandering Fatou components. It is shown that these constructions answer two questions posed in [OS16].

Contents

1. Introduction

From the dynamical viewpoint, an entire function $f : \mathbb{C} \to \mathbb{C}$ partitions the plane into two sets. There is the *Fatou set*, denoted $\mathcal{F}(f)$, that consists of the points where the family $(f^n)_{n\geq 1}$ is normal. And there is the *Julia set* - the complement of the Fatou set, denoted $\mathcal{J}(f)$. The Fatou set is open, and its components are called the *Fatou components* of f. The Fatou components are the regions of the plane where the dynamics of f are non-chaotic.

It is not difficult to see that $\mathcal{F}(f)$ is invariant under iteration by f. If we denote U as a component of the Fatou set, it is natural to study the behavior of the forward iterates $f^n(U)$. We use the following definition: U is called *periodic* if $f^n(U) \subseteq U$ for some n, and *preperiodic* if U is eventually mapped into a periodic component. On the other hand U is called a *wandering domain* if $f^n(U) \cap f^m(U) = \emptyset$ over all $n \neq m$.

Dennis Sullivan proved in [Sul85] that wandering domains do not occur for polynomials. On the other hand for more general entire functions, wandering domains are known to exist. We call a function $f : \mathbb{C} \to \mathbb{C}$ *transcendental* if f is entire but is not a polynomial. The first example of a transcendental function with a wandering domain was in fact produced before Sullivans' result - this was given by Baker in [Bak76].