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Pointwise estimates of global small solutions to the generalized double dispersion equation

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ABSTRACT

In this paper, we investigate the initial value problem for the generalized double dispersion equation in $\mathbb{R}^n$. Under suitable conditions, global classical solutions are proved for $n \geq 3$ and $n \geq 1$, respectively. Moreover, pointwise estimates of global classical solutions are established when $n \geq 3$ and n is an odd integer by Fourier splitting frequency technique.

1. Introduction

We investigate global classical solutions and pointwise estimates of global classical solutions to the generalized double dispersion equation

$$u_{tt} - \Delta u_{tt} - \nu \Delta u_t - \mu^2 \Delta u + \Delta^2 u = \Delta f(u) \tag{1.1}$$

with the initial value

$$t = 0 : u = u_0(x), \quad u_t = u_1(x). \tag{1.2}$$

Here $u = u(x,t)$ is the unknown function of $x \in \mathbb{R}^n$ and $t > 0$, $\mu > 0, \nu > 0$ are constants. The nonlinear term f is a smooth function with $f(u) = O(u^2)$.

Taking into account the possibility of energy exchange through lateral surfaces of the waveguide and using the Hamilton principle, the authors in [11] and [12] have derived the double dispersion equation

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$$v_{tt} - v_{xx} = \frac{1}{4}(cv^3 + 6v^2 + av_{tt} - bv_{xx} + dv_t)_{xx}. \tag{1.3}$$

Here v is proportional to the strain $\frac{\partial U}{\partial x}$ with U being the longitudinal displacement, and $a > 0, b > 0$ and $d \neq 0$ are some constants depending on the Young modulus, the shear modulus, the density of waveguide, and the Poisson coefficient. Owing to the scaling transformation $v(x,t) = u(y,\tau)$ with $y = \frac{1}{\sqrt{a}}x$ and $\tau = \frac{\sqrt{b}}{a}t$, (1.3) may be rewritten as

$$u_{\tau\tau} - u_{\tau\tau yy} + u_{yyyy} - \frac{d}{\sqrt{b}}u_{yy\tau} - \frac{a}{b}u_{yy} = \frac{a}{4b}(cu^3 + 6u^2)_{yy}. \tag{1.4}$$

It is not difficult to see that (1.1) is the generalized form of (1.4).

The existence, nonexistence and decay estimates of global solutions to (1.1) have attracted lots of researchers' interests and many interesting results have been established, we refer to [2,1,6,7,10,15,17,16,21, 20,22–24] and [25]. Chen, Wang and Wang [2] investigated the initial boundary value problem and obtained and existence and nonexistence of global solutions. The existence and nonexistence of global generalized solutions and classical solutions to periodic boundary value problem and Cauchy problem were established in [1]. On one hand, for the existence and nonexistence of global solutions to the initial value problem, we may refer to [15,10] and [21]. On the other hand, for asymptotic behavior of global solutions to the initial value problem, rich results have been established. For a class special initial data, decay estimates of global solutions are obtained by Wang and Da [17]. Recently, the decay results in [17] have been refined by Kawashima and Wang [7] when $u_0 \in H^m \cap L^1$, $u_1 \in H^{m-1} \cap \dot{W}^{-1,1}$ and $\|u_0\|_{H^{m+1}\cap L^1} + \|u_1\|_{H^m \cap \dot{W}^{-1,1}} (m \geq [n/2]+1)$ is small. Moreover, when $n \geq 2$, they also showed that our solution u can be approximated by the solution u_L to the linearized problem. But in one space dimension, the linear approximation failed. Fortunately, Kato, Wang and Kawashima [6] established a nonlinear approximation result to global solutions. Moreover, they showed that as time tends to infinity, the solution approaches the superposition of nonlinear diffusion waves which are given explicitly in terms of the self-similar solution of the viscous Burgers equation. Very recently, Wang and Chen [16] proved that asymptotic profile of solutions to the corresponding linear equation of (1.1) is given by the convolution of the fundamental solutions of heat and free wave equation. Moreover, applications of asymptotic profile are also given. But when the index of the Sobolev spaces is $n/2$, global existence and optimal decay estimates of solutions to (1.1), (1.2) are still open problems.We shall investigate this problem in future.

Attractor is an important topic in the research studies of the asymptotic behavior of the dissipative equation. Global attractor and exponential attractor of (1.1) and related equation have been established in [22–24] and [25]. Their results imply that (1.1) has some important constructions. More precisely, the global attractor of (1.1) represents the permanent regime that can be observed when the excitation starts from any point in natural energy space, and its dimension represents the number of degree of freedom of the related turbulent phenomenon and thus the level of complexity concerning the flow.

In this paper, our goal is to establish pointwise estimates of global classical solutions to the generalized double dispersion equation (1.1). To the best of our knowledge, this is a challenging open problem and few results are available. More precisely, we have the following two purposes. Firstly, under some suitable conditions, we prove that the problem (1.1), (1.2) has a unique global classical small solution for $n \geq 3$ and $n \geq 1$, respectively. Moreover, the decay estimate of global classical solutions is also established. For the details, we refer to Theorem 3.5 and Theorem 3.6. Secondly, when $n \geq 3$ is odd, we establish the pointwise estimate of global classical solutions obtained in Theorem 3.5. For the detail, we refer to Theorem 5.3. The proof of Theorem 5.3 relies on the pointwise estimate of solution operator to (1.1) in Theorem 4.8.

The plan of the paper is as follows. Firstly, make the detail analysis for solution operators to (1.1) in Section 2. Section 3 is devoted to prove the existence and decay estimate of global classical solutions to the problem (1.1), (1.2). Pointwise estimate of solution operators to (1.1) are established in Section 4. Finally, pointwise estimate of global classical solutions to the problem (1.1), (1.2) is discussed in Section 5.