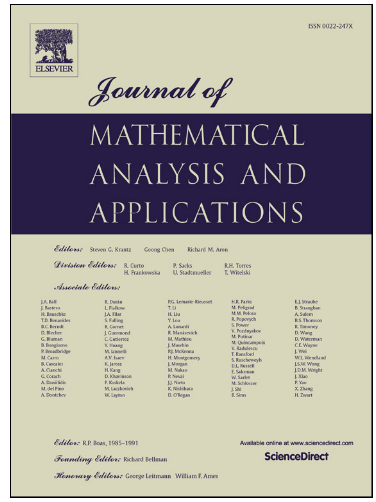

A note on periodic points of dendrite maps and their induced maps

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A NOTE ON PERIODIC POINTS OF DENDRITE MAPS AND THEIR INDUCED MAPS

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Abstract. Let X be a compact metric space and 2^X be the hyperspace of all nonempty closed subsets of X endowed with the Hausdorff metric. It is well known that for each continuous map $f : X \to X$, the density of periodic points of f implies the density of periodic points of the induced map $2^f : 2^X \to 2^X$. Méndez conjectured in [Topology Proceeding, 35 (2010), 281-290] that the converse is true when the phase space X is a dendrite. In [Discrete Contin. Dyn. Syst. **35** (2015), 771-792], Ŝpitalský constructed a continuous transitive map $F : X \to X$ on a dendrite X with only two periodic points. We prove in this note that the set of periodic points of its induced map 2^F is dense in 2^X. This answers in the negative Méndez's conjecture.

1. Introduction

A *continuum* is a compact connected metric space. An arc is any topological space homeomorphic to the unit interval $[0, 1]$. A *dendrite* is a locally connected continuum that contains no simple closed curves. Every subcontinuum of a dendrite is a dendrite ([7], Theorem 10.10) and every connected subset of X is arcwise connected ([7], Proposition 10.9). In addition, any two distinct points x, y of a dendrite X can be joined by a unique arc with endpoints x and y which is denoted by $[x, y]$. Denote $[x, y) = [x, y]\setminus\{y\}$, $(x, y] = [x, y] \setminus \{x\}$, and $(x, y) = [x, y] \setminus \{x, y\})$, respectively. If X is a dendrite and $p \in X$, then the *order* of p, denoted by $ord_X(p)$, is the number of connected components of $X \setminus \{p\}$. If $ord_X(p) = 1$, we say that p is an *endpoint* and if $ord_X(p) > 2$ then we say that p is a *branch point*. Denote by $\mathrm{E}(X)$ and $\mathrm{B}(X)$ the sets of endpoints and branch points of X respectively. A point $x \in X \setminus \mathrm{E}(X)$ is called a *cut point*. Denote by $\mathrm{Cut}(X)$ the set of cut points of X. It is known that $\mathrm{Cut}(X)$ is dense in X ([5], VI, Theorem 8, p.302).

If A is an arc in X with endpoints a and b, then A is said to be a *free arc* in X if the set $A \setminus \{a, b\}$ is open in X.

For a continuum X, let $\mathcal{FA}(X) = \cup\{Int_X(J) : J \text{ is a free arc in } X\}$. The continuum X is said to be *almost meshed*, if the set $\mathcal{FA}(X)$ is dense in X. The class of almost meshed continua contains dendrites with closed set of endpoints and dendrites with countable set of endpoints.

Let X be a compact metric space and $f : X \to X$ be a continuous map.

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