

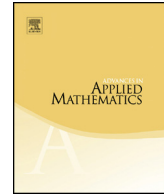


ELSEVIER

Contents lists available at ScienceDirect

Advances in Applied Mathematics

www.elsevier.com/locate/yaama



Volume inequalities for sections and projections of Wulff shapes and their polars [☆]



Ai-Jun Li ^{a,*}, Qingzhong Huang ^b, Dongmeng Xi ^c

^a School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo City 454000, China

^b College of Mathematics, Physics and Information Engineering, Jiaying University, Jiaying 314001, China

^c Department of Mathematics, Shanghai University, Shanghai 200444, China

ARTICLE INFO

Article history:

Received 11 September 2016

Received in revised form 25 May

2017

Accepted 28 May 2017

Available online xxxx

MSC:

52A40

Keywords:

Wulff shape

Isotropic measure

Support subset

ABSTRACT

Let $1 \leq k \leq n$. Sharp volume inequalities for k -dimensional sections of Wulff shapes and dual inequalities for projections are established. As their applications, several special Wulff shapes are investigated.

© 2017 Elsevier Inc. All rights reserved.

[☆] The first author was supported by NSFC–Henan Joint Fund (No. U1204102) and Key Research Project for Higher Education in Henan Province (No. 17A110022). The second author was supported in part by the National Natural Science Foundation of China (Nos. 11626115 and 11371239). The third author was supported by the National Natural Science Foundation of China (No. 11601310) and Shanghai Sailing Program (No. 16YF1403800).

* Corresponding author.

E-mail addresses: liaijun72@163.com (A.-J. Li), hqz376560571@163.com (Q. Huang), dongmeng.xi@live.com (D. Xi).

1. Introduction

Throughout, all Borel measures are understood to be nonnegative and finite. A convex body in \mathbb{R}^n is a compact convex set containing the origin in its interior. The polar body of a convex body K is given by $K^* = \{x \in \mathbb{R}^n : x \cdot y \leq 1 \text{ for all } y \in K\}$, where $x \cdot y$ denotes the standard inner product of x and y in \mathbb{R}^n . We use $\|\cdot\|$ to denote the Euclidean norm on \mathbb{R}^n . When A is a compact convex set in \mathbb{R}^n , we write $|A|$ for the volume of A in the appropriate subspace. Let $\text{supp } \nu$ denote the support of a measure ν and let P_H be the orthogonal projection onto a subspace H of \mathbb{R}^n .

Volume estimates for sections of convex bodies in \mathbb{R}^n are not easy, even in specific cases. For the cube $Q_n = [-\frac{1}{2}, \frac{1}{2}]^n$, Hensley [10] first showed that if H is a hyperplane of \mathbb{R}^n then $|H \cap Q_n|$ lies between 1 and 5, and conjectured that the upper bound is at most $\sqrt{2}$. This conjecture was solved by Ball [1,2], who also settled the more general case of k -dimensional sections.

The example of the regular simplex is much more complicated. Webb [25] proved that the maximal central hyperplane section is the one containing $n - 1$ vertices and the centroid. The question about the minimal central hyperplane section has not been completely solved yet. Brzezinski [7] proved a lower bound which differs from the conjectured minimal volume by a factor of approximately 1.27. For general k -dimensional sections, these questions were recently considered by Dirksen [8]. Other examples, such as ℓ_p^n -balls [5,6,20], complex cubes [22] and non-central sections of cubes [21], have also been investigated.

In this paper, we will study sections and projections of more general convex bodies than cubes and simplices. The main objects we consider are Wulff shapes [23], which were introduced by Wulff in 1901. Nowadays, it is an important notion in convex geometric analysis (see, e.g., [23]).

Definition. Suppose that ν is a Borel measure on S^{n-1} and that f is a positive, bounded, and measurable function on S^{n-1} . The *Wulff shape* $W_{\nu,f}$ determined by ν and f is defined by

$$W_{\nu,f} := \{x \in \mathbb{R}^n : x \cdot u \leq f(u) \text{ for all } u \in \text{supp } \nu\}. \tag{1.1}$$

The measure ν is said to be *even* if it assumes the same value on antipodal sets. When ν and f are both even, then $W_{\nu,f}$ is origin-symmetric. It is easy to see that $W_{\nu,f}$ is always convex and may be unbounded. In order for a k -dimensional subspace H of \mathbb{R}^n , to guarantee that $|W_{\nu,f}|$ and $|H \cap W_{\nu,f}|$ are finite, we consider Wulff shapes determined by measures ν which are isotropic and f -centered with respect to H . A Borel measure ν on S^{n-1} is called *isotropic* if

$$\int_{S^{n-1}} u \otimes u d\nu(u) = I_n, \tag{1.2}$$

Download English Version:

<https://daneshyari.com/en/article/5775385>

Download Persian Version:

<https://daneshyari.com/article/5775385>

[Daneshyari.com](https://daneshyari.com)