Advances in Applied Mathematics 88 (2017) 26–61

Contents lists available at ScienceDirect

Advances in Applied Mathematics

www.elsevier.com/locate/yaama

Limit theory for the Gilbert graph

Matthias Reitzner [a,*], Matthias Schulte [b], Christoph Thäle [c]

[a] *Institute of Mathematics, Osnabrück University, Germany*
[b] *Institute of Mathematical Statistics and Actuarial Science, University of Bern, Switzerland*
[c] *Faculty of Mathematics, Ruhr University Bochum, Germany*

ARTICLE INFO

Article history:
Received 1 June 2016
Accepted 30 December 2016
Available online 26 January 2017

MSC:
primary 60D05
secondary 60F05, 60F10, 60G55

Keywords:
Central limit theorem
Compound Poisson limit theorem
Concentration inequality
Covariogram
Gilbert graph
Malliavin–Stein method
Poisson point process
Random geometric graph
Stable limit theorem
Talagrand's convex distance

ABSTRACT

For a given homogeneous Poisson point process in $\mathbb{R}^d$ two points are connected by an edge if their distance is bounded by a prescribed distance parameter. The behavior of the resulting random graph, the Gilbert graph or random geometric graph, is investigated as the intensity of the Poisson point process is increased and the distance parameter goes to zero. The asymptotic expectation and covariance structure of a class of length-power functionals are computed. Distributional limit theorems are derived that have a Gaussian, a stable or a compound Poisson limiting distribution. Finally, concentration inequalities are provided using the convex distance.

© 2017 Elsevier Inc. All rights reserved.

* Corresponding author.
E-mail addresses: matthias.reitzner@uos.de (M. Reitzner), matthias.schulte@stat.unibe.ch (M. Schulte), christoph.thaele@rub.de (C. Thäle).

http://dx.doi.org/10.1016/j.aam.2016.12.006
0196-8858/© 2017 Elsevier Inc. All rights reserved.

1. Introduction

Let η_t be a homogeneous Poisson point process of intensity $t > 0$ in a compact convex observation window $W \subset \mathbb{R}^d$ with volume $V(W) > 0$ and let $(\delta_t : t > 0)$ be a sequence of positive real numbers such that $\delta_t \to 0$, as $t \to \infty$. A random graph $G(\eta_t, \delta_t)$ is defined by taking the points of η_t as its vertices and by connecting two distinct points $x, y \in \eta_t$ by an edge if and only if

$$0 < \|x - y\| \leq \delta_t \,,$$

where $\|x - y\|$ stands for the Euclidean distance between x and y. The resulting graph $G(\eta_t, \delta_t)$ is called Gilbert graph, random geometric graph or distance graph, and in the special cases $d = 1$ and $d = 2$ also interval or disc graph, respectively. It has been introduced (in the planar case) by Gilbert [12] in 1961. As opposed to the Erdős–Rényi random graph, which is a purely combinatorial object, the Gilbert graph is a random *geometric* graph because in its construction the relative position of the points in space plays an essential rôle.

The aim of the present paper is to investigate functionals related to the edge lengths of the Gilbert graph. The length-power functionals $L_t^{(\tau)}$ of interest are defined by

$$L_t^{(\tau)} := \frac{1}{2} \sum_{(x,y)\in\eta_{t,\neq}^2} \mathbb{1}(\|x - y\| \leq \delta_t)\, \|x - y\|^\tau \,, \tag{1.1}$$

where $\tau \in \mathbb{R}$ and $\eta_{t,\neq}^2$ stands for the set of all pairs of distinct points of η_t. The cases $\tau = 0$ and $\tau = 1$ are of particular importance. Namely, $L_t^{(0)}$ is the number of edges of $G(\eta_t, \delta_t)$ and $L_t^{(1)}$ is its total edge length. In our analysis, we focus on the asymptotic behavior of $L_t^{(\tau)}$, as $t \to \infty$ and $\delta_t \to 0$. For this situation we first compute the asymptotic expectation of $L_t^{(\tau)}$ and thereby establish a connection to the covariogram of the underlying convex set W. We also analyze the asymptotic covariances of $L_t^{(\tau_1)}$ and $L_t^{(\tau_2)}$ for $\tau_1, \tau_2 > -d/2$. In a next step, we develop an understanding for the asymptotic behavior of the length-powers of the individual edges. In this context, we will show that the collection of all edge length-powers converges, after a suitable re-scaling, to a Poisson point process on the real line. We then develop a comprehensive distributional limit theory for the functionals $L_t^{(\tau)}$ for all powers τ. Depending on the choice of τ and the distance parameters $(\delta_t : t > 0)$, we obtain central limit theorems as well as non-central limit theorems in which a stable or a compound Poisson limiting random variable shows up. These results provide a complete picture of the asymptotic distributional behavior of the length-power functionals $L_t^{(\tau)}$ and substantially add to the existing literature. Moreover, we shall also provide multivariate versions of the mentioned central and non-central limit theorems. Our main tool to prove the central limit theorems is the recently developed Malliavin–Stein method for Poisson functionals (see [19,20,27]). This technique allows us to provide rates of convergence in Kolmogorov distance for the range $\tau > -d/4$. For

Download English Version:

https://daneshyari.com/en/article/5775396

Download Persian Version:

https://daneshyari.com/article/5775396

Daneshyari.com