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The automorphism group of the s-stable Kneser graphs ☆

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A R T I C L E   I N F O

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A B S T R A C T

For $k, s \geq 2$, the s-stable Kneser graphs are the graphs with vertex set the k-subsets S of $\{1, \dots, n\}$ such that the circular distance between any two elements in S is at least s and two vertices are adjacent if and only if the corresponding k-subsets are disjoint. Braun showed that for $n \geq 2k + 1$ the automorphism group of the 2-stable Kneser graphs (Schrijver graphs) is isomorphic to the dihedral group of order $2n$. In this paper we generalize this result by proving that for $s \geq 2$ and $n \geq sk+1$ the automorphism group of the s-stable Kneser graphs also is isomorphic to the dihedral group of order $2n$.

1. Introduction

Given a graph G, $V(G)$, $E(G)$ and $\mathrm{Aut}(G)$ denote its vertex set, edge set and automorphism group, respectively. Let $[n] := \{1, 2, 3, \dots, n\}$. For positive integers n and k such that $n \geq 2k$, the *Kneser graph* $\mathrm{KG}(n, k)$ has as vertices the k-subsets of $[n]$ with edges defined by disjoint pairs of k-subsets. A subset $S \subseteq [n]$ is *s-stable* if any two of its

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elements are at least "at distance s apart" on the n-cycle, i.e. $s \leq |i-j| \leq n-s$ for distinct $i, j \in S$. For $s, k \geq 2$, we denote $[n]^k_s$ the family of s-stable k-subsets of $[n]$. The *s-stable Kneser graph* $\mathrm{KG}(n,k)_{s-\mathrm{stab}}$ [6,10] is the subgraph of $\mathrm{KG}(n,k)$ induced by $[n]^k_s$.

In a celebrated result, Lovász [5] proved that the chromatic number of $\mathrm{KG}(n,k)$, denoted $\chi(\mathrm{KG}(n,k))$, is equal to $n-2k+2$, verifying a conjecture due to M. Kneser [3]. After this result, Schrijver [7] proved that the chromatic number remains the same for $\mathrm{KG}(n,k)_{2-\mathrm{stab}}$. Moreover, this author showed that $\mathrm{KG}(n,k)_{2-\mathrm{stab}}$ is χ-critical. Due to these facts, the 2-stable Kneser graphs have been named *Schrijver graphs*. These results were the base for several papers devoted to Kneser graphs and stable Kneser graphs (see e.g. [1,4,6,8–10]). In addition, it is well known that for $n \geq 2k+1$ the automorphism group of the Kneser graph $\mathrm{KG}(n,k)$ is isomorphic to S_n, the symmetric group of order n (see [2] for a textbook account).

More recently, in 2010 Braun [1] proved that the automorphism group of the Schrijver graphs $\mathrm{KG}(n,k)_{2-\mathrm{stab}}$ is isomorphic to the dihedral group of order $2n$, denoted D_{2n}. In this paper we generalize this result by proving that the automorphism group of the s-stable Kneser graphs is isomorphic to D_{2n}, for $n \geq sk+1$.

Firstly, notice that if $n = sk$, the s-stable Kneser graph $\mathrm{KG}(n,k)_{s-\mathrm{stab}}$ is isomorphic to the complete graph on s vertices and the automorphism group of $\mathrm{KG}(n,k)_{s-\mathrm{stab}}$ is isomorphic to S_s.

From the definitions we have that D_{2n} injects into $\mathrm{Aut}(\mathrm{KG}(n,k)_{s-\mathrm{stab}})$, as D_{2n} acts on $\mathrm{KG}(n,k)_{s-\mathrm{stab}}$ by acting on $[n]$. Then, we have the following fact.

Remark 1.1. $D_{2n} \subseteq \mathrm{Aut}(\mathrm{KG}(n,k)_{s-\mathrm{stab}})$.

In the sequel, the arithmetic operations are taken *modulo* n on the set $[n]$ where n represents the 0. Let us recall an important result due to Talbot.

Theorem 1.2 *(Theorem 3 in [8]). Let n, s, k be positive integers such that $n \geq sk$ and $s \geq 3$. Then, every maximum independent set in $\mathrm{KG}(n,k)_{s-\mathrm{stab}}$ is of the form $\mathcal{I}_i = \{I \in [n]^k_s : i \in I\}$ for a fixed $i \in [n]$.*

For $n \geq sk+1$ we observe that $\{i, i+s, i+2s, \ldots, i+(k-1)s\}$ and $\{i, i+s+1, i+2s+1, \ldots, i+(k-1)s+1\}$ belong to $[n]^k_s$ for all $i \in [n]$. Then, we can easily obtain the following fact.

Remark 1.3. Let $n \geq sk+1$ and $i, j \in [n]$. If $i \neq j$, then $\mathcal{I}_i \neq \mathcal{I}_j$.

2. Automorphism group of $\mathrm{KG}(n,k)_{s-\mathrm{stab}}$

This section is devoted to obtain the automorphism group of $\mathrm{KG}(n,k)_{s-\mathrm{stab}}$. To this end, let us introduce the following graph family. Let n, s, k be positive integers such that $n \geq sk+1$. We define the graph $G(n,k,s)$ with vertex set $[n]$ and two vertices $i, j \in [n]$