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An efficient iterative updating method for hysteretic damping models

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ABSTRACT

Finite element model updating techniques are used to update the finite element model of a structure in order to improve its correlation with the experimental dynamic test data. This paper presents an efficient iterative method for finite element matrix updating problem in a hysteretic damping model based on a few of complex measured vibration modal data. By using the proposed iterative method, the unique symmetric solution can be obtained within finite iteration steps in the absence of roundoff errors by choosing a special kind of initial matrix triple. Some theorems are stated and proved, numerical results show that the presented method can be used to update finite element models to get better agreement between analytical and experimental modal parameters.

1. Introduction

Highly accurate analytical structural models are necessary for analyzing and predicting the dynamic performance of complex structures. By the finite element technique, an n degrees of freedom spatial model with structural damping can be described by the second order differential equation as

$$M_a\ddot{q}(t) + (K_a + iH_a)q(t) = 0, \tag{1}$$

where $i = \sqrt{-1}$, M_a, $K_a \in \mathbf{R}^{n\times n}$ and $H_a \in \mathbf{R}^{n\times n}$ are respectively analytical mass, stiffness, and hysteretic damping matrices, $q(t) \in \mathbf{R}^{n\times 1}$ is the displacement vector. In general, M_a, H_a and K_a are all real-valued symmetric. Eq. (1) is usually known as the finite element analytical model. Separation of variables $q(t) = xe^{\omega t}$ in Eq. (1), leads to structural eigenproblem:

$$\lambda M_a x + (K_a + iH_a)x = 0, \tag{2}$$

where $\lambda = \omega^2$.

With the development of computational mechanics and modern computer technology, the finite element method is useful for many applications in engineering practice such as structural response prediction, structural control, structural health monitoring, reliability and risk assessment, etc. [1–5]. But there are some inaccuracies or uncertainties that may be associated with a finite element model. The discretization error, arising due to the approximation of a continuous structure by a finite number of individual elements, is inherent to the finite element technique. While other inaccuracies may be due to the difficulties in the modeling of joints, boundary conditions and damping–the dynamic modelling of a structure often has

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to incorporate some kind of damping, in order to simulate adequately and accurately the real behaviour of the structure. The most widely used models for representing the damping are the viscous and the hysteretic ones. Idealization and simplification of structural details and lack of knowledge of exact material properties lead to a significant discrepancy may exist between the modal properties calculated by the constructed finite element model and those identified from the vibration measurements of the actual structure. Many investigations show that the differences between the numerical and experimental frequencies may exceed 10% or sometimes even 40% [5,6]. Nowadays, the appearances of high precision aircrafts and spacecrafts, large-scale bridges and other new engineering structures impose higher demands on the reliability and accuracy of their initial models [7]. The problem of how to modify an analytical model from the dynamic measurements is known as the model updating in structural dynamics. Finite element model updating is a procedure to identify and correct uncertain modelling parameters based on the experimental results that leads to better predictions of the dynamic behaviour of a target structure.

Model updating has been an active area of research for the last three decades and various methods have been developed for correcting analytical models to predict test results more closely. Interested readers are referred to survey papers [8,9]. A detailed theoretical analysis of model updating techniques can be found in the authoritative book [4]. For example, Baruch and Bar-Itzhack [10,11] obtained a closed-form solution of the updated stiffness matrix by using Lagrange multipliers for minimizing the changes in the stiffness matrix to satisfy specified constraints under the assumption that the mass matrix is exact. Baruch [12] and Berman and Nagy [13] developed the analytical model improvement procedures to update the mass matrix and stiffness matrix alternately. Wei [14,15] presented the formulations to correct both the mass and stiffness matrices based on constrained minimization theory. These early methods are direct and computationally efficient and mainly aim at undamped systems.

Due to the important contribution of damping on structural vibration, model updating of damped structures becomes significant. In recent years, the model updating problems for linear viscously damped elastic systems (or gyroscopic systems) and nonlinear damped systems have been considered by many authors (see [16–25], etc.). However, problems for updating hysteretic damping models have received little attention in these years. It is well known that the main difference between viscous and hysteretic damping models is that, for the viscous system, the energy dissipation per cycle depends linearly upon the frequency of oscillation, whereas for the hysteretic case it is independent of frequency. Although much progress has been made in recent years in the development of analytical procedures for evaluating the response of hysteretic damping models [26–30], very few authors pay attention to hysteretic damping model updating problems because the free vibration response for a system with hysteretic damping is necessarily complex, whereas the modal data of viscously damped elastic systems are closed under complex conjugation. In [31], authors provided an extended application of the constrained eigenstructure assignment method (CEAM), which was first introduced in [32], to finite element model updating. The existing formulation was modified to accommodate larger systems by developing a quadratic linear optimization procedure which is unconditionally stable. Further refinements included the updating of the mass matrix, a hysteretic damping model, and the introduction of elemental correction factors.

We observe that the iterative methods for finite element coefficient matrix updating have received little attention in these years. In this paper, we will develop a finite iterative updating method for finite element models with hysteretic damping which can incorporate the measured modal data into the initial analytical model to produce an adjusted finite element model on the mass, damping and stiffness matrices that closely match the experimental modal data. We believe that the method proposed can be applied to other types of dynamical systems (for example, [33,34]) with some suitable modifications.

Assume that M_a, K_a and H_a are all real-valued symmetric matrices, the problem of updating mass, stiffness and hysteretic damping matrices simultaneously can be mathematically formulated as following inverse eigenvalue problem and an associated optimal approximation problem.

Problem IEP. Let $\Gamma = \mathrm{diag}\{\gamma_1, \ldots, \gamma_p\} \in \mathbf{C}^{p\times p}$ and $Z = [z_1, \ldots, z_p] \in \mathbf{C}^{n\times p}$ be the measured eigenvalue and eigenvector matrices, where $p \ll n$. Find real-valued symmetric matrices M, H and K such that

$$MZ\Gamma + (K + iH)Z = 0. \tag{3}$$

Problem II. Let $\mathcal{S}_E$ be the solution set of Problem IEP. Find $(\hat{M}, \hat{H}, \hat{K}) \in \mathcal{S}_E$ such that

$$\|\hat{M} - M_a\|^2 + \|\hat{K} - K_a\|^2 + \|\hat{H} - H_a\|^2 = \min_{(M,H,K)\in\mathcal{S}_E} (\|M - M_a\|^2 + \|K - K_a\|^2 + \|H - H_a\|^2). \tag{4}$$

The paper is organized as follows. In Section 2, an efficient iterative method is presented to solve Problem IEP and Problem II, and several properties of Algorithm 1 are proved. By using the proposed iterative method, the minimum Frobenius norm solution of Problem II can be obtained by choosing a special kind of initial matrix triple. In Section 3, a numerical example is used to test the effectiveness of the proposed algorithm. Some concluding remarks are given in Section 4.

Throughout this paper, we shall adopt the following notation. $\mathbf{C}^{m\times n}$ and $\mathbf{R}^{m\times n}$ denote the set of all $m \times n$ complex and real matrices, and the set of all symmetric matrices in $\mathbf{R}^{n\times n}$ by $\mathbf{SR}^{n\times n}$. $A^\top$, $\mathrm{tr}(A)$ and $\mathcal{R}(A)$ stand for the transpose, the trace and the column space of the matrix A, respectively. I_n represents the identity matrix of order n. For $A, B \in \mathbf{R}^{m\times n}$, an inner product in $\mathbf{R}^{m\times n}$ is defined by $(A, B) = \mathrm{tr}(B^\top A)$, then $\mathbf{R}^{m\times n}$ is a Hilbert space. The matrix norm $\|\cdot\|$ induced by the inner product is the Frobenius norm. Given two matrices $A = [a_{ij}] \in \mathbf{R}^{m\times n}$ and $B \in \mathbf{R}^{p\times q}$, the Kronecker product of A and B is defined by $A \otimes B = [a_{ij}B] \in \mathbf{R}^{mp\times nq}$. Also, for an $m \times n$ matrix $A = [a_1, a_2, \ldots, a_n]$, where $a_i, i = 1, \ldots, n$, is the ith column