Applied Mathematics and Computation 320 (2018) 116–130

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

State estimation for stochastic discrete-time systems with multiplicative noises and unknown inputs over fading channels

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ARTICLE INFO

Keywords:
Robust estimation
Multiplicative noise
Unknown input
Fading channel
Unbiased minimum-variance filter

ABSTRACT

This paper deals with robust state estimation problem for a class of stochastic discrete-time systems with multiplicative noises and unknown inputs over fading channels. An unbiased unknown input insensitive filter is designed such that the variance of the estimation error is minimized in the sense of the so-called $\mathcal{P}$-estimation. The filter gain matrix is derived through solving a recursive Riccati equation and a generalized Lyapunov equation. A necessary and sufficient condition that guarantees the existence of the filter is given, which establishes a fundamental limit on the mean square capacity of each fading channel. Unknown input estimation and finite horizon stability of the proposed filter are also discussed. To illustrate the effectiveness of the proposed approach, the proposed algorithm is applied to a faulty remote controlled uninterruptible power system, where both the state and the fault are estimated.

1. Introduction

During the past decades, due to increasing applications in real world, such as bias compensation, fault detection and isolation, and calculating key factors in geophysical and environmental applications, state estimation subject to unknown inputs has attracted intensive attention. Generally speaking, there are mainly two fundamental approaches to solve this problem. One is to formulate the problem into an unbiased minimum-variance filtering (UMVF) framework, where a specific constraint on the filter gain matrix is involved. In this case, the estimated state is decoupled from the exogenous inputs via modified Luenberger filter, or unknown input Kalman filter (UIKF), see for example, [1–7]. The other equivalent methodology is the descriptor Kalman filtering method through employing the least-squares data fitting (LSDF) or the maximum likelihood (ML) techniques, where both the states and unknown inputs are simultaneously estimated [8,9].

On another front line, in the literature of robust control and filtering, fault detection and isolation, finance, biology, and economics, the modelling uncertainties which are often presented as multiplicative noises (or named as stochastic uncertainties) may severely degrade the performance of the system. In real world applications, this kind of state dependent multiplicative systems arise naturally from random failures and repairs of the components, changes in the interconnections

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https://doi.org/10.1016/j.amc.2017.09.008

of subsystems, sudden environment changes, and modifications on the operating point of a linearized model of nonlinear systems [10,11]. Therefore, fruitful efforts are devoted to this field [12–19], and accordingly, some research attention has been paid to systems with multiplicative noise for filtering problems in both H_2 and H_∞ contexts, e.g., [11,20–24]. Especially speaking, with the extensive utilization of wireless sensor networks in military surveillance, environmental monitoring, health care, and building automation, system may suffer from intermittent measurements, uncertain sensing or unreliable communication. These limited capacities over transiting process can also be appropriately modelled by multiplicative noises, and such unreliable transmitting channels are usually nominated as the multiplicative memoryless fading channels, which cover complex fading channel structures, such as uncertain channel with random packet dropout, delayed and erased channel, channel with erasure and holder, *N*th-order Rice fading channel, as well as output channel with sensor gain degradation [25–28]. A great number of contributions have been dedicated to filtering problems with fading transmitted data. We refer to [21–24,29] and references therein.

In recent years, robust filtering problems subject to unknown inputs, multiplicative noise and unreliable channels have emerged in the area of networked control systems (NCS) and cyber-physical systems (CPS) [30–33]. The unknown inputs in such scenarios can describe interconnecting external inputs in the context of large scale NCS, actuator faults that affect remote controllers, as well as cyber attacks by using CPS, such as denial-of-service (DoS) attacks, deception attacks, replay attacks and covert attacks. Notice that multiplicative noises that introduced by stochastic model uncertainties or unreliable measurement links brings some new challenges for UIKF design, which indeed results from the unavoidable calculation of the second moment of the state variables. In case that there is no prior information of the unknown inputs, this second moment of the state is not available, and the aforementioned works in [1–9] cannot be directly applied for robust filter design. To tackle these challenges of filtering problems, some compromise has been made by restricting some special assumptions on the systems. For example, in discrete-time setting, [30] investigated robust filtering problem subject to intermittent unknown inputs by assuming that the intermittent indicators are online available. Nevertheless, the derived algorithm fails to work in case that the indicators are unknown. [31,32] proposed a least square filtering methodology for estimating state and constant unknown input with random delayed measurements, but the proposed approach may not work efficiently for time-varying unknown inputs. [33] provided a UIKF design scheme for systems with missing measurement by using the state estimation to compute the second moment of the state, while this technique is not available for systems without direct-feed through term. In continuous-time setting, [34] proposed a $\mathcal{P}$-estimation method for bilinear systems with multiplicative noises and unknown inputs. The idea behind this method is to split the state into two parts, where the first component is a function of the observations while the second component is estimated. However, by assuming that the unknown input terms can be eliminated through linear transformation technique, the existence conditions of the filter given in [34] seem conservative. To overcome these defects, more efforts should be made to derive an efficient filtering algorithm for simultaneously considering stochastic uncertainty, unknown input, and channel fading, which motives our present study.

In this paper, we aim to propose a novel robust filtering algorithm for stochastic discrete-time systems with multiplicative noises and unknown inputs over fading channels. The contribution of this paper lies in four-folds: (1) Without giving additional prerequisite knowledge of the unknown inputs or restricting special assumptions on the system, an extension version of the $\mathcal{P}$-estimation methodology is proposed, which guarantees a robust unbiased minimum-variance estimation. (2) A necessary and sufficient condition on the existence of the proposed filter is derived in accordance with the probability distribution of the fading channels, which indicates an explicit lower bound on the mean square capacity of transmitting observations for $\mathcal{P}$-estimation. (3) A recursive algorithm for calculating the filter gain matrix is provided through solving a Riccati equation and a Lyapunov equation. (4) The stability of the proposed filter in finite horizon is analyzed, and a sufficient condition for this stability is established.

Notations: Throughout this paper, for a matrix X, X^{T} and X^{-1} stand for the transpose and inverse of X, respectively. $X>0 (X<0)$ means X is positive (negative) definite. $\mathrm{tr}(X)$ denotes the trace of a square matrix X. $\lambda_i(X)$ and $\rho(X)$ define the ith eigenvalue and the spectral radius of a symmetric matrix X. $diag\{X_1, X_2, \ldots, X_n\}$ represents a block diagonal matrix with diagonal blocks $X_1, X_2, \ldots, X_n$. R^n means the set of n-dimensional real vectors and $R^{m\times l}$ represents the set of $m\times l$-dimensional real matrices. I and 0 denote identity matrix and zero matrix with appropriate dimensions, respectively. Given a random variable ϑ, $\mathrm{E}\{\vartheta\}$ means the expectation of ϑ. $\mathrm{Prob}\{\mathcal{A}\}$ represents the probability of an event $\mathcal{A}$. The symbols '$\circ$' and '$\otimes$' represent the Hadamard product and Kronecker product, respectively.

2. Problem statement

Consider the following discrete stochastic system

$$\begin{cases} x(k+1) = Ax(k) + \alpha(k)A_\alpha x(k) + Gd(k) + w(k) \\ \quad y(k) = \Xi(k)Cx(k) + \upsilon(k) \end{cases} \tag{1}$$

where $x(k)\in R^n$, $y(k)\in R^{n_y}$, and $d(k)\in R^{n_d}$ denote the state, measurement output, and unknown input, respectively; $w(k)\in R^n$ and $\upsilon(k)\in R^{n_y}$ are process noise and measurement noise, respectively. $A\in R^{n\times n}$, $C\in R^{n_y\times n}$ and $G\in R^{n\times n_d}$ are known constant matrices. $\Xi(k) = diag\{\gamma_1(k), \ldots, \gamma_{n_y}(k)\}$ describes the memoryless multiplicative fading phenomenon in each output channel. $\{\alpha(k)\}$ is a scalar wide sense stationary, second-order process, which is usually named as multiplicative noise to describe the model uncertainties.