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Stabilization of stochastic delay systems via a disordered controller☆

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ABSTRACT

In this paper, the stabilization for stochastic delay systems is achieved by a disordered controller. Different from the traditionally stabilizing controllers, the controller designed here experiences a disorder between control gains and system states. By exploiting the robust method, the above disorder is described as a controller having special uncertainties. Moreover, the probability distribution of such uncertainties is embodied by a Bernoulli variable. A sufficient condition for the existence of such a disordered controller is given with LMIs, where the probability is considered in its design procedure. Based on this description, a more general but complicated case where the corresponding probability is not exact but has a uncertainty is further studied, whose LMI conditions are presented too. Finally, a numerical example is exploited to demonstrate the effectiveness and superiority of the proposed methods.

1. Introduction

As we know, time delay is commonly encountered in variously practical systems. It also leads to many negative effects such as oscillation, instability and poor performance. During the past decades, a lot of research topics of time-delayed systems have been considered, such as stability analysis [1–6], stabilization [7–14], dissipativity analysis [15] and dissipative and passive control [16–18], output control [19–21], H_∞ control and filtering [22–31], state estimation [32–34], synchronization [35–38], slide control [39–41], positivity analysis [42], and so on.

By investigating the most results on system synthesis in literature, it is seen that there were few references to consider the disordering problem including time delay systems. As we know, the introduction of the shared communication networks has great advantages such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. However, the motivation of disordering problem also comes from the data transmitted through the shared communication networks. It is an inevitable phenomenon that the transmitted data arriving at the destination is usually out of order [43–45]. In other words, the transmission of data packets cannot always satisfy the “first send first arrive”

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principle. Instead, the newest control signal may arrive at the destination before the older one. Packet disorder could increase the difficulty of system modeling and make its analysis and synthesis complicated. It also has negative impacts on system performance and could lead to waste of resources. Up to now, there are very few results to study this issue, where some new interesting and challenging problems are introduced. By exploiting a packet disordering compensation method, some LMI conditions were presented in reference [46]. Based on transforming the underlying system into a discrete-time system with multi-step delays, the stability and H_∞ control problems of NCSs with packet disordering were considered in [47,48], while some less conservative results were given in [49]. Based on the average dwell-time method, a kind of packet reordering method was proposed in [50]. By investigating these references, it is seen that the originally studied systems are all without any time delay. To our best knowledge, very few results are available to design a disordered controller for delay systems. All the facts motivate the current research.

In this paper, the stabilization problem of stochastic delay systems closed by a disordered controller is considered. The main contributions of this paper are summarized as follows: 1) Different from references [7,8,11,13] where no disorder occurs, a generally kind of stabilizing controller experiencing a disorder between control gains and system states is proposed for stochastic delay systems; 2) Compared with the disordered stabilization results [46–50], the original system considered in this paper is more general and has time delay. Moreover, the disorder considered here takes place between system states and control gains, which is different from the above references; 3) In contrast to the above methods dealing with disorder, such a disorder is modeled into a controller with special uncertainties and handled by applying a robust approach. More importantly, the switching probability between such uncertainties is also taken into account in the controller design; 4) Because of the results presented with LMI forms, they could be applied to many different and complicated situations such as discrete-time systems, signal estimation, and so on.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices. $\mathscr{E}\{\cdot\}$ means the mathematical expectation of $[\cdot]$. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, diag $\{\cdots\}$ for a block-diagonal matrix, and $(M)^\star \triangleq M+M^T$.

2. Problem formulation

Consider a kind of stochastic delay systems described as

$$\begin{cases} dx(t)=(Ax(t)+A_\tau x(t-\tau)+Bu(t))dt+(Cx(t)+C_\tau x(t-\tau)+Du(t))d\omega(t) \\ x(t)=\phi(t), \quad t\in[-\tau,0] \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state, $u(t)\in\mathbb{R}^m$ is the control input, and $\omega(t)$ is an one-dimensional Brownian motion or Wiener process. Matrices A, A_τ, B, C, C_τ, and D are known matrices of compatible dimensions. Time delay τ satisfies $\tau\geq 0$. $\phi(t)$ is a continuous function and defined from $[-\tau,0]$ to $\mathbb{R}^n$. It is known that the traditional state feedback controllers for delay systems are commonly as follows:

$$u(t)=Kx(t) \tag{2}$$

$$u(t)=K_\tau x(t-\tau) \tag{3}$$

$$u(t)=Kx(t)+K_\tau x(t-\tau) \tag{4}$$

where K and K_τ are control gains to be determined. It is said that controller (4) is more general and has some advantages. The main reason is both delay and non-delay states are taken into account. However, the action of controller (4) needs an assumption that the control gains and theirs related states should be available in a right sequence. Unfortunately, due to some practice constraints, this assumption may be very hard satisfied. In this paper, a kind of controller experiencing a disorder phenomenon is proposed and described by

$$u(t)=\begin{cases} Kx(t)+K_\tau x(t-\tau), & \text{no disordering} \\ K_\tau x(t)+Kx(t-\tau), & \text{disordering occuring} \end{cases} \tag{5}$$

It is rewritten to be

$$\begin{aligned} u(t) &= [\alpha(t)K+(1-\alpha(t))K_\tau]x(t)+[\alpha(t)K_\tau+(1-\alpha(t))K]x(t-\tau) \\ &= [K_\tau+\alpha(K-K_\tau)+(\alpha(t)-\alpha)(K-K_\tau)]x(t)+[K+\alpha(K_\tau-K)+(\alpha(t)-\alpha)(K_\tau-K)]x(t-\tau) \end{aligned} \tag{6}$$

Here, $\alpha(t)$ is the Bernoulli variable and assumed to take values in a finite set $\mathbb{S}\triangleq\{0,1\}$, one has

$$\begin{aligned} &\Pr\{\alpha(t)=1\}=\alpha, \quad \Pr\{\alpha(t)=0\}=1-\alpha \\ &\Pr\{\alpha(t)-\alpha\}=0, \quad \Pr\{(\alpha(t)-\alpha)^2\}=\alpha(1-\alpha) \end{aligned} \tag{7}$$

Then, controller (6) is equivalent to

$$u(t)=[\hat{K}+(\alpha(t)-\alpha)\Delta\hat{K}]x(t)+[\hat{K_\tau}+(\alpha(t)-\alpha)\Delta\hat{K_\tau}]x(t-\tau) \tag{8}$$