Applied Mathematics and Computation 314 (2017) 299–309

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Global Output-Feedback Adaptive Stabilization for Planar Nonlinear Systems with Unknown Growth Rate and Output Function☆

Xuehua Yan[a,*], Xinmin Song[b], Zhonghua Wang[a], Yun Zhang[a]

[a] School of Electrical Engineering, University of Jinan, Jinan 250022, Shandong, PR China
[b] School of Information Science and Engineering, Shandong Normal University, Jinan 250014, Shandong, PR China

ARTICLE INFO

Keywords:
Nonlinear systems
Unknown output function
Adaptive control
Global stabilization
Output-feedback

ABSTRACT

This paper studies the problem of global output-feedback stabilization by adaptive method for a class of planar nonlinear systems with unmeasurable state dependent growth and unknown output function. It is worth emphasizing that not only the growth rate, but also the upper and lower bounds of the derivative of output function are not required to be known in the paper. To solve the problem, we propose a new adaptive output-feedback control scheme based on only one dynamic high-gain, by skillfully constructing the new Lyapunov function, and flexibly using the ideas of universal control and backstepping. It is shown that the state of the closed-loop system is bounded while global asymptotic stability can be achieved. Two examples including a practical example are given to demonstrate the effectiveness of the theoretical results.

1. Introduction

Control design problem of nonlinear systems has been a topic of active research in control field [1–29]. Global output-feedback stabilization problem of nonlinear systems with linearly unmeasured states dependent growth has captured a great deal of attention during the last two decades. A large number of stabilizing results have been obtained for uncertain nonlinear systems under different assumptions [1–17]. However, the common phenomenon existing in [3–17] is that the system output y and the state component x_1 always keep or can become linear relationship. Although the system output in [1,2] is unknown C^1 function $h(x_1)$, not only the growth rate, but also the upper and lower bounds of the derivative of the output function, are assumed to be known constants. More limitedly, among these existing work, expect [3], control coefficients themselves or at least one of the upper and lower bounds of control coefficients/the derivative of output function is required to be known. Recently, to relax the aforementioned restriction, [3] utilizes time-varying output-feedback control scheme for a class of nonlinear systems with unmeasurable states dependent time-varying growth and unknown control coefficients. One of the main contribution in [3] is that no known constants are supposed on the lower or upper bound of

☆ This work is supported in part by the National Science Foundation of Shandong Province (No. ZR2016FM17), the Excellent Young Scholars Research Fund of Shandong Normal University, the National Natural Science Foundation of China (No. 61304013, 61403162), and the Doctoral Foundation of University of Jinan (No. XBS1413).

* Corresponding author.
E-mail addresses: cse_yanxh@ujn.edu.cn (X. Yan), xinminsong@sina.com (X. Song), cse_wzh@ujn.edu.cn (Z. Wang), cse_zhangyun@ujn.edu.cn (Y. Zhang).

http://dx.doi.org/10.1016/j.amc.2017.07.014
0096-3003/

constant control coefficients. Whereas, due to the presence of time-varying gain, the proposed time-varying controller in [3] is not easy to implement from a practical standpoint.

Adaptive control is a conventional and effective method to deal with the unknowns/uncertainness existing in the system. Under the premise of the unmeasurable state dependent growth, [9–14] propose the corresponding adaptive control scheme based on dynamic high-gain scaling for systems whose uncertain nonlinear functions have output dependent growth rate. By constructing suitable adaptive high-gain, [6–9] study the global stabilization problem for uncertain nonlinear systems with unknown growth rate and/or unknown control coefficients. However, control coefficients are precisely known in [8–14], and [6,7] require the known information on both the upper and lower bounds of unknown control coefficients. For planar nonlinear system, by adopting a dynamic high-gain K-filter, [4,5] further relax the assumption on unknown control coefficients and only require that their lower bounds are known. How to completely remove the restriction on the upper and lower bounds of control coefficients (more generally, the derivative of output function) by adaptive technique is a very interesting problem and still remains unsolved, and as stated in [3], when the serious unknowns exist in control coefficients/output function, an adaptive technique is much more difficult to carry out. Based on the above-mentioned discussion, a problem naturally appears: *for the system with unknown control coefficients (generally unknown output function), whose (derivative's) upper and lower bounds are unknown, whether adaptive method can be applicable?* To the best of the authors' knowledge, it is still an open question.

Mainly motivated by [4–6], by flexibly using the methods of universal control and backstepping, this paper designs a new, relatively simple adaptive output-feedback controller for a class of planar nonlinear systems with unknown growth rate, as well as unknown upper and lower bounds of the derivative of unknown output function. Concretely, compared with the closely related works [1–6], the highlights of the paper can be summarized as follows: First, by adaptive technique, rather than time-varying approach, the derivative of the unknown output function is not required to possess known upper and lower bounds. Second, by skillfully introducing the dynamic high-gain into the stabilizing function and Lyapunov function, we effectively handle the unknowns/uncertainties in the system nonlinearities and output function. Based on the introduced dynamic high-gain observer, a new adaptive output-feedback controller is constructed to guarantee that global asymptotic stability is achieved. Third, compared with the most closely related work [4], this paper only makes use of one dynamic high-gain, but need not add the additional update law to eliminate the negative effect due to unknown upper bounds of control coefficients, and moreover, we adopt a high-gain observer, rather than high-dimensional K-filter, in the stabilizing control design, which reduces the dimension of the closed-loop system.

The outline of the paper is organized as follows. Section 2 formulates the system model and the control objective. In Section 3, we propose a new global output-feedback stabilizing control scheme. Section 4 summarizes the main results of the paper. In Section 5, we give two examples to illustrate the effectiveness of the theoretical results. Finally, Section 5 states some concluding remarks.

2. System model and control objective

2.1. System model

Consider the stabilizing control problem for a class of planar nonlinear systems in the following form:

$$\begin{cases} \dot{x}_1 = x_2 + \phi_1(t,\, x,\, u), \\ \dot{x}_2 = u + \phi_2(t, x, u), \\ y = h(x_1), \end{cases} \tag{1}$$

where $x = [x_1, x_2]^{\mathrm{T}} \in \mathbf{R}^2$ is the system state vector with the initial value $x_0 = x(0)$; $u \in \mathbf{R}, y \in \mathbf{R}$, are the control input and system output, respectively; $\phi_i : \mathbf{R}^+ \times \mathbf{R}^2 \times \mathbf{R} \to \mathbf{R},\ i = 1,\ 2$ are unknown functions but continuous in the first argument and locally Lipschitz in the rest arguments; h: $\mathbf{R} \to \mathbf{R}$ are unknown C^1 function satisfying $h(0) = 0$, and its derivative $\dot{h}$ is of known sign. Without loss of generality, we assume that $\dot{h}$ is always positive. In what follows, suppose only the system output is measurable.

To obtain the desired objective of the paper, the system (1) is imposed the following assumptions.

Assumption 1. There exists an unknown positive constant θ, such that

$$\begin{aligned} &|\phi_1(t, x, u)| \le \theta |x_1|, \\ &|\phi_2(t, x, u)| \le \theta(|x_1| + |x_2|) \end{aligned} \tag{2}$$

Assumption 2. There exist two unknown positive constants $\underline{l}$ and $\bar{l}$ such that

$$\underline{l} \le \dot{h} \le \bar{l}. \tag{3}$$

It is not difficult to see that the systems described by (1) include those with unknown control coefficients such as [4] described by the following general form

$$\begin{cases} \dot{\eta}_1 &= g_1\eta_{i+1} + \psi_i(t,\, \eta,\, u), \\ \dot{\eta}_2 &= g_2 u + \psi_2(t,\, \eta,\, u), \\ y &= \eta_1, \end{cases} \tag{4}$$