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New criteria of stability analysis for generalized neural networks subject to time-varying delayed signals[☆]

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ABSTRACT

This paper focuses on the new criteria of stability analysis for generalized neural networks (GNNs) subject to time-varying delayed signals. A new methodology is employed with the aids of slack variables. By constructing an augmented Lyapunov–Krasovskii functional (LKF) involving Newton–Leibniz enumerating and triple integral term, some less conservative conditions are achieved in terms of linear matrix inequality (LMI). Numerical examples including real-time application are given to illustrate the superiority and effectiveness of proposed approach.

1. Introduction

Over the past few decades, neural networks (NNs) have been attracted much attention because of its widely applications in various areas, such as control, image processing, signal processing and so on. It is noted that dynamic behaviors of NNs are inevitable determine its potential application, especially the equilibria of NNs [6–8]. Time delay frequently occur in the NNs and it always causes severe performance degradation or even instabilities of systems. Therefore, the stability analysis of NNs with time delays in the practical systems are important challenges [30–35]. Up to now, all kinds of method have been introduced to investigate the time-delayed NNs and considerable results on the topic of NNs with time delays have been reported in the literature [1–5].

On the other hand, the stability criteria of NNs are split into two classes, for example, delay-dependent stability criteria [9–15] and delay-independent ones [16]. It is well known that delay-dependent ones is a key point in reducing conservatism of stability conditions when compared with delay-independent ones. For example, new criterion has been constructed for dealing with time-varying delays systems in [9,10]. Therefore, many researchers have drawn their attention to deal with various issues of delay-dependent NNs and many significant discovers have been reported [17–23].

To reduce the conservatism of delay-dependent stability conditions, appropriate LKFs are developed, which are involving more information of time delays and activation functions. The augmented LKFs approaches have been play a role of crucial impression for deriving the less conservativeness criteria, such as delay-partitioning LKFs [24], discretized LKFs [25,26] and

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augmented LKFs [28]. Furthermore, bounding the derivative of LKFs also can lessening of conservatism of the results. For example, the free-weighting matrices plays an important role in injecting additional variables [22,23], the Jensen's inequalities [26,29], the Wirtinger's inequalities [20] and the free-matrix-based inequalities [29]. The problems of stability analysis have been discussed via LKF were mostly conservative [27,28]. Therefore, there is room for further research.

Motivated by the ahead of examination, this main goal of this paper is to establish less conservatism criteria via an augmented LKF involving the triple integral term. The tighter bounds for the derivative of LKF is obtained with the help of Wirtinger's and Jensen's inequalities, which plays a crucial role in conservatism reduction. New stability criteria is displayed in the sorts of LMIs, in which some slack matrices are employed to provide new degrees of freedom. The proposed GNN is more general and the results achieved in the paper have less conservatism with the developed new methodologies.

Notation: R^n denotes the $n-$dimensional Euclidean space; the superscripts -1 and T denote the inverse and transpose, respectively. $\cdot$ denotes the expectation operator with respect to some probability measure; the symbol $\mathbf{sym}(\mathcal{Q})$ is used to represent $\mathcal{Q}+\mathcal{Q}^\mathsf{T}$; $*$ is employed to represent a term that is induced by symmetry.

2. Preliminaries

Consider the following GNN:

$$\dot{x}(t) = -Ax(t) + W_0 g(Wx(t)) + W_1 g(Wx(t-\tau(t))) + J \tag{1}$$

for $t \geq 0$, where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^\mathsf{T} \in \mathcal{R}^n$ is the state vector; $g(x(t)) = [g_1(W(x_1(t))), g_2(W(x_2(t))), \ldots, g_n(W(x_n(t)))]^\mathsf{T} \in \mathcal{R}^n$ and $J = [J_1, J_2, \ldots, J_n]^T \in \mathcal{R}^n$ respectively represents the neuron activation function and input vector; $A = diag\{a_1, a_2, \ldots, a_n\}$ stands for a diagonal matrix with $a_i > 0$; W, W_0 and W_1 are the connection weight matrix and the delayed connection weight matrix, respectively. The time-varying delay $\tau(t)$ satisfies

$$0 \leq \tau(t) \leq \tau \tag{2}$$

$$\dot{\tau}(t) \leq \mu \tag{3}$$

where τ and μ are constants.

In this paper, the neuron activation function is assumed satisfies

$$k_i^- \leq \frac{g_i(\phi) - g_i(\varphi)}{\phi - \varphi} \leq k_i^+, \phi \neq \varphi, i = 1, 2, \ldots, n \tag{4}$$

where k_i^-, k_i^+ are known real constants. If there exists an equilibrium point x^*, the equilibrium point of (1) can be shift to the origin

$$z(t) = x(t) - x^*, f(Wz(t)) = g(Wz(t) + Wx^*) - g(Wx^*). \tag{5}$$

The system GNN (1) can be rewritten as

$$\dot{z}(t) = -Az(t) + W_0 f(Wz(t)) + W_1 f(Wz(t-\tau(t))) \tag{6}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^\mathsf{T}$, $f(W(z(t))) = [f_1(W(z_1(t))), f_2(W(z_2(t))), \ldots, f_n(W(z_n(t)))]^\mathsf{T}$.

It is clear from (4) that

$$k_i^- \leq \frac{f_i(W_i z_i(t))}{W_i z_i(t)} \leq k_i^+, i = 1, 2, \ldots, n. \tag{7}$$

Let $k_i = \max\{|k_i^-|, |k_i^+|\}$, then

$$-k_i \leq \frac{f_i(W_i z_i(t))}{W_i z_i(t)} \leq k_i, i = 1, 2, \ldots, n. \tag{8}$$

Denote $L_1 = diag\{k_1^-, k_2^-, \ldots, k_n^-\}$, $L_2 = diag\{k_1^+, k_2^+, \ldots, k_n^+\}$, $L_3 = diag\{k_1, k_2, \ldots, k_n\}$ and $W = diag\{W_1, \ldots, W_n\}$

Lemma 2.1. *[19] Let z be a differentiable function: $[\alpha, \beta] \to \mathcal{R}^n$. For symmetric positive matrices $\mathcal{R}$,and $\mathcal{N}_i \in \mathcal{R}^{4n\times n}$, one has:*

$$-\int_\alpha^\beta \dot{z}^T(s) R \dot{z}(s) ds \leq \vartheta^T \Omega \vartheta,$$

where

$$\Omega = (\beta-\alpha)(\mathcal{N}_1 \mathcal{R}^{-1} \mathcal{N}_1^T + \frac{1}{3}\mathcal{N}_2 \mathcal{R}^{-1} \mathcal{N}_2^T + \frac{1}{5}\mathcal{N}_3 \mathcal{R}^{-1} \mathcal{N}_3^T) + Sym\{\mathcal{N}_1 \Xi_1 + \mathcal{N}_2 \Xi_2 + \mathcal{N}_3 \Xi_3\}$$

$$\Xi_1 = e_1 - e_2,\ \Xi_2 = e_1 + e_2 - 2e_3,\ \Xi_3 = e_1 - e_2 - 6e_3 + 6e_4$$

$$\vartheta = \begin{bmatrix} z^T(\beta) & z^T(\alpha) & \frac{1}{\beta-\alpha}\int_\alpha^\beta z^T(s)ds & \frac{2}{(\beta-\alpha)^2}\int_\alpha^\beta \int_\alpha^s z^T(u)duds \end{bmatrix}^\mathsf{T}$$