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A Decoupled method for image inpainting with patch-based low rank regulariztion☆

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ABSTRACT

In this paper, we propose a decoupled variational method for image inpainting in both image domain and transform domain including wavelet domain and Fourier domain. The original image inpainting problem is decoupled as two minimization problems with different energy functionals. One is image denoising with low rank regularization method, i.e., the patch-based weighted nuclear norm minimization (PWNNM). The other is linear combination in image domain or transform domain. An iterative algorithm is then obtained by minimizing the two problems alternatingly. In particular, we derive the variational formulas for PWNNM and reformulate the denoising process into three steps: image decomposition, patch matrix denoising, and image reconstruction. The convergence of the numerical algorithm is proved under some assumptions. The numerical experiments and comparisons on various images demonstrate the effectiveness of the proposed methods.

1. Introduction

Image inpainting is an important topic in computer vision and image processing. The problem occurs when the observed data is incomplete in the sense that some pixels or coefficients of the target image under certain transform are missing or corrupted [1,37]. The aim of image inpainting is to recover an ideal image from the incomplete data. Here the ideal image is expected to have edges, structures and texture patterns consistent with the given data in a natural way for human eyes [4].

Image inpainting can be classified into two classes: image domain inpainting and transform domain inpainting. The former means that some pixels in the image domain are missing, while the latter means that some coefficients in certain transform domain are missing. Image domain inpainting has widely applications in restoring ancient drawings and old pictures, where some pixels are missing or damaged due to aging or scratch, or in removal of objects in photography or films for special effects [1,11]. In many practical applications, since images are formatted, transmitted, stored or encoded as transformed coefficients of images in some transformed domain, the coefficients may be lost or corrupted and thus it leads to the transform domain inpainting problem. The widely used image transforms include discrete cosine transform (DCT) (e.g. Joint Photographic Experts Group (JPEG) image), wavelet transform (e.g. JPEG 2000 image), and Fourier transform (e.g. magnetic resonance (MR) imaging) [8,10], etc.

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In recent years, many useful techniques have been developed to solve the image domain inpainting problem. These methods can be roughly classified into pixel based methods and exemplar based methods. In the pixel based methods, the missing region is filled by diffusing the image information from the known region to the missing region pixel by pixel via some partial differential equations (PDEs) [1,2,13,28,29], or by updating the sparse representation coefficients of the image under certain transforms such as wavelet, tight frame or dictionary [4,14,16,19,23,27,38]. In the exemplar based inpainting methods, the missing region is filled by propagating the image information in the known region patch by patch [11,35,36].

The transform domain inpainting problem has also been widely studied using variational methods, especially wavelet domain inpainting and Fourier domain inpainting. In the variational methods, the regularization scheme plays a leading role. Chan et al. [9] propose to fill in the missing coefficients in the wavelet domain by the total variation (TV) minimization method and evolving the associated PDE numerically, which is relatively slow. To speed up, many fast numerical methods designed for the TV denoising problem are applied to solve the transform domain inpainting problem and some efficient algorithms are obtained. A fast optimization transfer algorithm (OTA) is proposed by Chan et al. [7], in which Chambolle's fast dual projection algorithm [6] is used to solve the TV denoising subproblem. Later Chan et al. [8] propose to use the alternating direction method (ADM) to solve the TV wavelet inpainting model. The latter is more efficient. A primal-dual type numerical algorithm is proposed by Wen et al. [33] to solve the TV wavelet inpainting problem and the convergence is proved. Another primal-dual hybrid gradient method is proposed by Ye and Zhou [37]. Zhang and Chan [40] propose to use the nonlocal TV (NLTV) regularization in wavelet inpainting instead of TV, which greatly improves the image quality than the TV based methods for texture images.

The Fourier domain inpainting problem has been widely addressed in the MR imaging problem that is also termed as Compressed Sensing. Goldstein and Osher [17] propose to use the TV regularization in the Fourier domain inpainting problem and develop a fast numerical scheme based on the split Bregman method. Chen et al. [10] propose two fast algorithms based on the primal-dual hybrid gradient method. Ma et al. [26] propose a model with both TV and wavelet regularization, and derive an efficient numerical algorithm using the operator splitting technique. Guo et al. [20] use both total generalized variation (TGV) and shearlet as regularization terms. A fast numerical algorithm is derived based on the alternating direction method of multiplier (ADMM). The NLTV regularization is adopted by Zhang et al. [39], and an efficient algorithm is proposed using the Bregmanized operator splitting technique. Li and Zeng [24] propose a promising decoupled method for both wavelet and Fourier transform domain inpainting based on the BM3D filter [12].

Image inpainting can also be regarded as a matrix completion problem since the image or its coefficients under certain transform are usually stored as a digital matrix. Low rank matrix completion has attracted considerable interest recently [3,5,22,30,31,34]. As nuclear norm is the convex surrogate of the rank function of matrices, it is widely used as a regularization term in the low rank matrix completion problem. Cai et al. [3] propose the singular value thresholding (SVT) algorithm for the nuclear norm minimization problem. Accelerating numerical methods are proposed in [30,31,34]. However, since most images are not low rank, these low rank based matrix completion methods usually can not produce satisfactory results for the image inpainting problem. Hu et al. [22] propose a new truncated nuclear norm regularization which works well for image inpainting. Generally speaking, there are many nonlocal similar patches in a natural image. Hence the matrix obtained by stacking the nonlocal similar patch vectors should be a low rank matrix and has sparse singular values. Based on this observation, Gu et al. [18] propose a weighted nuclear norm minimization (WNNM) method which works on local patches for image denoising. It is reported that the WNNM method outperforms many state-of-the-art denoising algorithms including BM3D.

The contribution of this paper is two-fold. Firstly, we derive the matrix representation for patch-based WNNM (PWNNM) decomposition and reconstruction operators. Then we reformulate the PWNNM denoising process into three basic steps: image decomposition, patch matrix denoising, and image reconstruction. With these integrated formulation, PWNNM can be regarded as a regular regularization method and can be used in many image processing problems. Secondly, we propose a new method using PWNNM regularization for both image domain inpainting and transform domain inpainting. The main idea is to decouple the original problem into two alternating steps: PWNNM denoising and linear combination in image domain or transform domain. Different from the existing coupled variational methods in which only one energy functional is essentially involved, the proposed method minimize two different energy functionals alternatingly. One advantage of this decoupled method is that the original problem is split into two relatively independent step, such that we have more flexibility in the choice of method for each step. The experimental results demonstrate that the proposed method is competitive with the state-of-the-art inpainting algorithms in terms of PSNR index and visual quality.

The remainder of this paper is organized as follows. In Section 2, we derive the variational formulation of PWNNM. In Section 3, we propose our decoupled method for image inpainting and study the convergence of the iterative algorithm. In Section 4, we display some experimental results and comparisons to illustrate the effectiveness of our method. Finally, we conclude the paper in Section 5.

2. The variational formulation of PWNNM

The PWNNM method is proposed in [18] for image denoising. The main idea is to apply WNNM on many matrices formed by similar patches in the image. This method is very effective especially for high level Gaussian noise. In this section, we derive the variational formulation of PWNNM which includes three steps: image decomposition, patch matrix denoising and image reconstruction. This formulation has not been presented in the existing work so far as we know.