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Study of predictor corrector block method via multiple shooting to Blasius and Sakiadis flow

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ABSTRACT

In this paper, a predictor corrector two-point block method is proposed to solve the well-know Blasius and Sakiadis flow numerically. The Blasius and Sakiadis flow will be modeled by a third order boundary value problem. The main motivation of this study is to provide a new method that can solve the higher order BVP directly without reducing it to a system of first order equation. Two approximate solutions will be obtained simultaneously in a single step by using predictor corrector two-point block method able to solve the third order boundary value problem directly. The proposed direct predictor corrector two-point block method will be adapted with multiple shooting techniques via a three-step iterative method. The advantage of the proposed code is that the multiple shooting will converge faster than the shooting method that has been implemented in other software. The developed code will automatically choose the guessing values in order to solve the given problems. Some numerical results are presented and a comparison to the existing methods has been included to show the performance of the proposed method for solving Blasius and Sakiadis flow.

1. Introduction

Blasius equation is one of the basic equations of fluid dynamics and first introduced by Blasius [1] in 1908. This equation describes the velocity profile of the fluid in the boundary layer theory on a half infinite interval. The Blasius equation is modeled by the nonlinear two-point boundary value problem

$$\begin{aligned} & f''' + \frac{f''f}{2} = 0 \\ & f(0) = \alpha, \quad f'(0) = \gamma, \quad \lim_{\eta\to\infty} f'(\eta) = \beta. \end{aligned} \tag{1}$$

There are no available closed-form solution of Blasius equation. Many methods or techniques have been used to obtain the analytical and numerical solutions for this equation. The first analytic solution for solving Blasius equation have been

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proposed by Blasius [1] as follow:

$$f'''(\eta) = \sum_{k=0}^{\infty}\left(-\frac{1}{2}\right)^k \frac{A_k\sigma^{k+1}}{(3k+2)!}\eta^{3k+2}, \text{ where } A_0 = A_1 = 1 \text{ and}$$
$$A_k = \sum_{r=0}^{k-1}\binom{3k-1}{3r}A_rA_{k-r-1},\ k \geq 2.$$

Ahmad and Al-Barakati [2] provided an approximate analytic solution of the Blasius equation by modified the 4/3 Pade approximant. Wickramasinghe [3] studied the comparison between decomposition and numerical solution profiles of Blasius equation. Adomou et al. [4] had used adomian methods to get the analytical solution of Blasius equation. A comparison among the homotopy perturbation method and the decomposition method with the variational iteration method for solving Blasius equation have been studied by Gholaminejad et al. [5].The first numerical solution for solving Blasius equation was provided by Howarth [6] in 1938. Recently, Xu and Guo [7] applied the fixed point method to obtain the semi-analytical solution to Blasius equation.

In this paper, we discuss two flows of Blasius equation subject to different boundary conditions. The first type of flow we focus is Blasius flow. The boundary condition of Blasius flow is as follow:

$$f(0) = 0, \quad f'(0) = 0, \quad \lim_{\eta\to\infty} f'(\eta) = 1. \tag{2}$$

In the Blasius flow, the fluid motion is produced by the free stream. When the plate moves with constant velocity in a calm fluid, we called it Sakiadis flow and was first introduced by Sakiadis [8]. The boundary condition of Sakiadis flow

$$f(0) = 0, \quad f'(0) = 1, \quad \lim_{\eta\to\infty} f'(\eta) = 0. \tag{3}$$

Pantokratoras [9] and Hady et al. [10] provided a theoretical study of the effect of variable fluid properties on the classical Blasius and Sakiadis flow.

The aim of this research is to propose a new algorithm based on block method to solve the Blasius equation with Blasius and Sakiadis flow directly using multiple shooting techniques with three-step iterative method. The three-step iterative method had been proposed by Yun [11] to solve the nonlinear equation and it has been proved that this method has fourth-order convergence. The initial guessing values in the developed code will be chosen automatically and it is much easier compare to the guessing value using in Maple where user need to guess the values very close to the exact. Hence, the advantages of the proposed method managed to give accurate and faster results in terms of execution time compared to Maple and Matlab solver.

2. Predictor corrector two-point block method

2.1. Derivation of predictor corrector two-point block method

The predictor corrector two-point block method is used to develop a numerical algorithm for computing two approximate solutions directly of the third order ODE. The general third order ODE is as follows:

$$y''' = f(x, y, y', y''). \tag{4}$$

The derivation of the block method is based on the derivation introduced in Majid et al. [12]. The approximate solutions are divided into series of blocks with each block containing two points as shown in Fig. 1. The two approximate values y_{n+1} and y_{n+2} with step size h are computed simultaneously in a block using the three back values at the point x_n, x_{n-1} and x_{n-2}. The approximate solution y_{n+1} and y_{n+2} can be obtained by integrating Eq. (4) once, twice and thrice.

Integrate Once:

$$y''(x_{n+1}) - y''(x_n) = \int_{x_n}^{x_{n+1}} f\left(x, y, y', y''\right)dx$$
$$y''(x_{n+2}) - y''(x_n) = \int_{x_n}^{x_{n+2}} f\left(x, y, y', y''\right)dx. \tag{5}$$

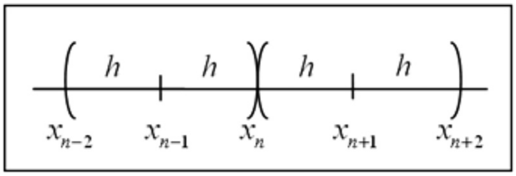

Fig. 1. Two-point block method.