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Short Communication

Hierarchy of stability criterion for time-delay systems based on multiple integral approach☆

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ABSTRACT

Taking a class of time-delay systems as research object, this brief aims at developing a theoretical support on the hierarchy of stability criterion which is derived by the multiple integral approach and free-weighting matrix technique. The hierarchy implies that the conservatism of stability criterion can be reduced by increasing the ply of integral terms in Lyapunov–Krasovskii functional (LKF). Together with three numerical experiments, the hierarchy of stability criterion is further shown.

1. Introduction

Time-delay is frequently encountered in practical systems, and it is also one of the main sources leading to instability, oscillations, or other poor performance of a system [1–3]. Therefore, the stability analysis of time-delay systems has provoked considerable attention. Accordingly, many renewed approaches have been reported, such as the free-weighting matrix approach [4–7], the delay partitioning approach [8–12], and the integral inequality-based approach [13–25], relaxed Lyapunov functional approach [26–28], flexible terminal approach [29,30], to name a few.

Recently, an approach, titled multiple integral approach, has been proposed to analyze the stability of time-delay systems in [31]. It is asserted in [31] that the stability criteria are less and less conservative with the increasing of the ply of integral terms in Lyapunov–Krasovskii functional (LKF). The similar assertion is also presented in [32–36]. However, the assertion is merely clarified by numerical experiments rather than the relevant theoretical basis. To the best of our knowledge, such basis has not been provided in the literatures, which motivates the current study.

The main purpose of this brief is to show that the multiple integral approach is a reliable way to reduce the conservativeness of stability criterion from a theoretical perspective. In this brief, the multiple integral forms of LKF and free-weighting matrix technique are employed to develop a new stability criterion for a class of linear time-delay systems. Strict mathematical proof shows that the developed stability criterion forms a hierarchy of LMI indexed by the ply of integral terms, which means that increasing the ply of integral terms in LKF helps to reduce the conservatism of the stability criterion. Therefore, results of the brief provide a theoretical support for those results in [31–36].

Notations: Throughout this brief, $\mathbb{N}^+$ and $\mathbb{R}^n$ denote the set of positive integer and the n-dimensional Euclidean space, respectively. $\mathbb{R}^{n\times m}$ and $\mathbb{R}_+^{n\times n}$ stand for the set of all $n\times m$ real matrices and the set of all $n\times n$ real symmetric positive definite matrices, respectively. The superscripts '-1' and 'T' represent the inverse and transpose of a matrix, respectively.

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The asterisk '$*$' is used to denote a matrix that can be inferred by symmetry. For real symmetric matrix X, $X > 0$ ($X \geq 0$) means that X is positive definite (respectively, semidefinite); $Sym\{X\} = X + X^T$. I_n denotes the n-dimensional identity matrix. $0_{n\times m}$ denotes the $n \times m$-dimensional zero matrix.

2. Main results

Consider the following linear time-delay system [13–16,33]:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau) + A_D \int_{t-\tau}^{t} x(s)ds, & \forall\ t \geq 0, \\ x(t) = \phi(t), & \forall\ t \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, A, A_d, and $A_D \in \mathbb{R}^{n\times n}$ are known and constant matrices. The initial condition, $\phi(t)$, is a continuous vector-valued function defined over $[-\tau, 0]$, and the delay $\tau > 0$ is a scalar.

For system (1), we develop the following stability criterion.

Proposition 2.1. *For a given $m \in \mathbb{N}^+$ and a scalar $\tau > 0$, the time-delay system (1) is asymptotically stable if there exist matrices $P_m \in \mathbb{R}_+^{mn\times mn}$, $Q \in \mathbb{R}_+^{n\times n}$, $R_r \in \mathbb{R}_+^{n\times n}$ ($r = 1, 2, \ldots, m$), and any matrices $N_j \in \mathbb{R}^{(m+1)n\times n}$ ($j = 0, 1, \ldots, m-1$) such that the following LMI holds:*

$$\Sigma_m(\tau) = \begin{bmatrix} \Phi_0 & \tau N_0 & \frac{\tau^2}{2!}N_1 & \cdots & \frac{\tau^m}{m!}N_{m-1} \\ * & -\tau R_1 & 0 & \cdots & 0 \\ * & * & -\frac{\tau^2}{2!}R_2 & \cdots & 0 \\ * & * & * & \ddots & \vdots \\ * & * & * & * & -\frac{\tau^m}{m!}R_m \end{bmatrix} < 0, \tag{2}$$

where

$$\begin{aligned} \Phi_0 =& Sym\left\{ \Pi_1 P_m \Pi_2^T + \sum_{r=1}^{m} I_{(m+1)n} N_{r-1} \left(\frac{\tau^{r-1}}{(r-1)!} e_1 - e_{r+1} \right)^T \right\} \\ & + e_1 Q e_1^T - e_2 Q e_2^T + \sum_{r=1}^{m} \frac{\tau^r}{r!} \Psi R_r \Psi^T, \\ \Pi_1 =& \begin{bmatrix} e_1 & e_3 & e_4 & \cdots & e_{m+1} \end{bmatrix}, \\ \Pi_2 =& \begin{bmatrix} \Psi & e_1 - e_2 & \tau e_1 - e_3 & \cdots & \frac{\tau^{m-2}}{(m-2)!} e_1 - e_m \end{bmatrix}, \\ \Psi =& e_1 A^T + e_2 A_d^T + e_3 A_D^T, \\ e_i^T =& \begin{bmatrix} 0_{n\times(i-1)n} & I_n & 0_{n\times(m+1-i)n} \end{bmatrix}, \qquad i = 1, 2, \ldots, m+1. \end{aligned}$$

Proof. Define

$$\begin{aligned} \omega_0(t) &= \int_{t-\tau}^{t} x(s)ds, \\ \omega_r(t) &= \int_{-\tau}^{0}\int_{\theta_1}^{0} \cdots \int_{\theta_{r-1}}^{0}\int_{t+\theta_r}^{t} x(s)ds d\theta_r d\theta_{r-1} \cdots d\theta_1, \\ & \qquad\qquad\qquad r = 1, 2, \ldots, m-2, \\ \omega(t) &= \begin{bmatrix} \omega_0^T(t) & \omega_1^T(t) & \cdots & \omega_{m-2}^T(t) \end{bmatrix}^T, \\ \eta(t) &= \begin{bmatrix} x^T(t) & x^T(t-\tau) & \omega^T(t) \end{bmatrix}^T. \end{aligned}$$

Consider the following LKF with multiple integral terms:

$$V(t) = V_1(t) + V_2(t), \tag{3}$$

where

$$\begin{aligned} V_1(t) &= X^T(t) P_m X(t) + \int_{t-\tau}^{t} x^T(s) Q x(s) ds, \\ V_2(t) &= \sum_{r=1}^{m} Y_r(t), \end{aligned}$$

with $X(t) = \begin{bmatrix} x^T(t) & \omega^T(t) \end{bmatrix}^T$, and

$$Y_r(t) = \int_{-\tau}^{0}\int_{\theta_1}^{0} \cdots \int_{\theta_{r-1}}^{0}\int_{t+\theta_r}^{t} \dot{x}^T(s) R_r \dot{x}(s) ds d\theta_r d\theta_{r-1} \cdots d\theta_1.$$