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Detection of thermal bridges from thermographic images by means of image processing approximation algorithms

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A B S T R A C T

In this paper, we develop a procedure for the detection of the contours of thermal bridges from thermographic images, in order to study the energy performance of buildings. Two main steps of the above method are: the enhancement of the thermographic images by an optimized version of the mathematical algorithm for digital image processing based on the theory of sampling Kantorovich operators, and the application of a suitable thresholding based on the analysis of the histogram of the enhanced thermographic images. Finally, an improvement of the parameter defining the thermal bridge is obtained.

1. Introduction

The thermographic survey on the building envelope is a useful non-invasive method to detect thermal bridges that reduce the overall energy performance of buildings.

In [7,8] the above investigation has been performed by the help of a suitable index, the so-called *incidence factor of thermal bridge* I_{tb}, which points out the energetic incidence of a thermal bridge on the basis of the temperature decrement that it causes. The accuracy of this analysis depends on various aspects. One of the most important relies in the correct detection of the areas belonging to thermal bridges; such analysis described in [7,8] is strictly operator dependent. A further important aspect is given by the quality (i.e., the resolution) of the thermographic data available to perform the mentioned energetic analysis.

In the present paper, we introduce a segmentation procedure which allows to detect the thermal bridges from thermographic images, becoming the energy analysis of the buildings automatic and more accurate than the original one proposed in [7].

The algorithm developed in this paper is characterized by various steps, based on mathematical method of approximation theory and on techniques of Digital Image Processing (D.I.P.). Firstly, the thermographic images are reconstructed and enhanced in their resolution, by the application of the sampling Kantorovich (S-K) algorithm, see e.g., [24,25,31]. The latter method can be deduced from some approximation results concerning the theory of the well-known S-K operators, which has been deeply investigated in last years, see e.g., [29,30,32,33,48]. In particular, here we developed a suitable numerical

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optimized version of the S-K algorithm, useful to solve the application problem treated in this paper. Note that the S-K algorithm has been firstly introduced in [31], but its original implementation was not-optimized and required a longer CPU-time of execution.

The sampling Kantorovich operators represent in practice an L^1-version of the classical generalized sampling operators introduced by P.L. Butzer and his school at Aachen in the '80s, with the aim to obtain an approximate sampling formula. In some recent papers, it has been proved that the S-K operators are suitable for applications to real-world cases based on the study of suitable images; for instance, some models for the study of the behavior of buildings under seismic action have been successfully obtained by the applications of the S-K algorithm for the enhancement of thermographic images, see e.g., [24,25].

Here, the S-K algorithm for image enhancement has been implemented by a bivariate Jackson-type kernel, and the original resolution of thermographic images has been improved.

Then, by a probabilistic interpretation of the histogram of the pixels associated to the above enhanced thermographic images (see e.g., [40]), we determined a suitable threshold value which can be used in order to segment the thermal bridge.

The validation of the proposed method has been obtained from the experimental results effected in an ad-hoc built hot-box setup with controlled laboratory conditions (see e.g., [9,10]). More precisely, two types of bi-dimensional thermal bridges, with different shape, have been built and successively tested. The numerical results show that the proposed algorithm, besides identifying the geometry of the thermal bridge generated in walls composite by different materials, allows to improve the energy analysis of the buildings with respect to the original approach given in [7].

Indeed, such improvement has been validated by a comparison among the factor I_{tb}, firstly computed according with the data detected by the probes in the hot-box, the original procedure developed in [7], and the method here proposed, i.e., working with the thermographic image, enhanced by the S-K algorithm with the shape of the thermal bridge extracted by the automatic thresholding procedure.

The above numerical results show that, in thermal bridges caused by different materials, the method here introduced provides results closer to the most accurate approach, i.e., to those computed with the help of the probes, as numerically shown in Section 5.

2. Approximation by sampling Kantorovich operators and applications to image processing

In this section, we give a background concerning the main theoretical and applications aspects of the theory of S-K operators, see e.g., [11,54] for results concerning functions of one variable, and [30,32,33] for what concerns functions of several variables.

The above family of operators is typically used in approximation theory in order to reconstruct not-necessarily continuous signals, such as images, see [20,31,38], and they are useful for image reconstruction and enhancement, see [24,25]. We begin, by recalling the definition of the kernel functions used in order to define the above approximation process. In what follows, we will define as kernel any multivariate function $\chi : \mathbb{R}^n \to \mathbb{R}$, which satisfies the following conditions:

(χ1) χ is summable on $\mathbb{R}^n$, and bounded in a ball containing the origin of $\mathbb{R}^n$;
(χ2) For every $\underline{x} \in \mathbb{R}^n$:

$$\sum_{\underline{k}\in\mathbb{Z}^n} \chi(\underline{x}-\underline{k}) = 1;$$

(χ3) For some $\beta > 0$, we assume that the discrete absolute moment of order β is finite, i.e.,

$$m_\beta(\chi) := \sup_{\underline{u}\in\mathbb{R}} \sum_{\underline{k}\in\mathbb{Z}^n} |\chi(\underline{u}-\underline{k})| \cdot \|\underline{u}-\underline{k}\|^\beta \ < \ +\infty.$$

We immediately provide some typical examples of kernels which satisfy all the above assumptions (χ1), (χ2), and (χ3). The most used method to construct multivariate kernels is to consider the product of n kernels of one variable, see [18,26–28,56]. Indeed, for instance, the definition of the multivariate Fejér kernel can be formulated as follows:

$$\mathcal{F}_n(\underline{x}) = \prod_{i=1}^{n} F(x_i), \quad \underline{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n, \tag{1}$$

where $F(x)$, $x \in \mathbb{R}$, denotes the univariate Fejér kernel, which is defined by

$$F(x) := \frac{1}{2}\,\mathrm{sinc}^2\left(\frac{x}{2}\right), \quad x \in \mathbb{R}, \tag{2}$$

where the well-known *sinc*-function is that defined as $\sin(\pi x)/\pi x$, if $x \neq 0$, and 1 if $x = 0$, see e.g., [1,6,12,13,52,55]. By the *sinc*-function it is possible to define another class of kernels, which is widely used, i.e., the Jackson-type kernels, see e.g., [14,19,24,25,39]. The multivariate expression of the Jackson-type kernels (see, e.g., Fig. 1) is the following:

$$\mathcal{J}_k^n(\underline{x}) := \prod_{i=1}^{n} J_k(x_i), \quad \underline{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n, \tag{3}$$