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Sufficient regularity conditions for complex interval matrices and approximations of eigenvalues sets

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ABSTRACT

In this paper, two approaches are described to establish verifiable sufficient regularity conditions of complex interval matrices. In the first approach, a complex interval matrix is mapped to a real block interval matrix and then its sufficient regularity conditions are obtained. In the second approach, a necessary condition for the singularity of a complex interval matrix is derived and used to get its sufficient regularity conditions. As an application, the above derived sufficient regularity conditions are used to investigate the location of the outer approximations of individual eigenvalue sets of complex interval matrices. Two algorithms are proposed and results obtained are compared with those obtained by earlier methods and Monte Carlo simulations. The advantages of these algorithms are that they can detect gaps in between the approximations of the whole eigenvalue sets. The second algorithm is very effective compared to the first algorithm from the computational time point of view. Several numerical examples and statistical experiments are worked out to validate and demonstrate the efficacy of our work.

1. Introduction

Many engineering applications require the computation of eigenvalue sets of real or complex interval matrices. For example, stability of both discrete and continuous dynamical systems in control theory heavily depend on the eigenvalue sets of system matrices. Schur stability and Hurwitz stability of the system matrices are investigated for them. Computing eigenvalue sets of an interval matrix is a complicated task and used throughout in mathematics, physics and computer science. They are derived based on singularity and regularity of interval matrices. An interval matrix is said to be singular if it contains a singular matrix and regular if no such matrix exists in it. Verifiable sufficient conditions for them to have approximation to eigenvalue sets are one of the most important and challenging areas of numerical analysis and scientific computing. This is essential to manage the uncertainty and inexactness due to various types of measurement or manufacturing error. For example, eigenvalues sets estimation of non-proportional damping structures with uncertainty or imprecision is very important in design and analysis of structures. They need availability of the choices of uncertainty model and quantity of information. Three types of approaches available to get them are probabilistic [1], Fuzzy set theory [2] and interval analysis [3,4]. The interval analysis approach uses the concept of intervals taking uncertainties in the form of compact intervals containing all possible solutions of the considered problems. Here, the uncertainties are taken as interval

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numbers, interval vectors or interval matrices. Interval analysis approach is most widely used in many fields of engineering and applied mathematics. The control theory [5] and stability analysis of interval dynamical systems [6] need to compute extremal eigenvalues of some complex interval matrices. Other practical applications are in the vibration systems [7], principal component analysis [8], robotics [9], etc.

The problem of regularity of real interval matrices is well studied [10–14]. It was introduced in [10] for its connection with the problem of computing the exact bounds on the solution of a system of n linear equations in n variables whose coefficients and right hand sides vary in some intervals. Some necessary and sufficient regularity conditions of real interval matrices are discussed there. In [11], it is shown that checking the regularity of interval matrix is an NP-hard problem. To characterize the regularity of an interval matrix, the notion of radius of non-singularity and a formula for its computation is given. For an interval matrix of order n, this formula requires the computation of spectral radius of 2^n real matrices. An upper bound for radius of non-singularity is given in [15]. A randomized approximation method using semi-definite relaxation is developed in [16] to give a tight bounds of radius of non-singularity with a constant relative error 0.7834. Other necessary and sufficient conditions for regularity of real interval matrices are introduced in [14] and shown to be NP-hard. However, these conditions are difficult to verify. To overcome this drawback, some computationally efficient sufficient conditions for regularity and singularity of real interval matrices are introduced in [12]. One of these sufficient conditions represents another class of interval matrices called strongly regular interval matrices. Some characterizations of strong regularity of interval matrices are studied in [17]. Since, checking regularity of interval matrices is a known NP-hard problem, a general algorithm for checking regularity, which is not a priori exponential, proposed in [13]. In [18], it is shown that a singular interval matrix contains a singular real matrix of very special form. Also, singularity of a real interval matrix is used to characterize its real eigenvalues sets. In recent years, new sufficient regularity condition is formulated in terms of positive definiteness of a certain point matrix, which generalize some of the earlier ones are introduced in [19]. Strong regularity conditions are generalized for parametric interval matrices are developed in [20,21]. A new regularity measure based on interval parametric linear programming is suggested in [22]. Real eigenvalue sets of real interval matrices can be well approximated using the regularity conditions of real interval matrices. Using sufficient regularity conditions, some fast computable iterative methods for outer approximation of eigenvalues sets of real interval matrices are presented in [23]. Given an outer approximation of the given eigenvalue set of an interval matrix, a filtering method that iteratively improves the approximation is proposed. The proposed method works for general as well as for symmetric interval matrices. In [24], the real eigenvalue bounds of generalized real interval eigenvalue problems are studied. The advantages achieved are that the computation procedure takes less time than earlier methods and can be applied for solving larger interval eigenvalue problems. An efficient subdivision algorithm is proposed in [25] using the algorithm given in [13] to approximate the eigenvalue sets of real interval matrices. Likewise, approximate bounds of the complex eigenvalue sets of a real interval matrix are studied in [26]. For a real interval matrix the entire eigenvalue sets is approximated by a rectangle enclosing them. Later, it was done for complex interval matrices in [27] by constituting some block real matrices. A cheap and tight bounds for the entire eigenvalue sets of a complex interval matrix as a rectangle in complex plane is provided in [28] and computed in terms of the extreme eigenvalue sets of a symmetric interval matrix. Another bounds of real and imaginary parts of the eigenvalue sets of a complex interval matrix are obtained by using weighted matrix measures in [29] by constructing four real matrices. A method for finding interval encloser of each eigenpair of a parametric interval matrix is developed in [30]. The problem of determining the exact (with in rounding errors) ranges for the real and imaginary parts of a real interval matrices with parametric dependencies is studied in [31]. It uses an iterative method which solves two quadratic systems in each iteration. Constant sign conditions of eigenvectors and positive quadratic system is needed for this method. The main motivation and novelty of our work is to achieve gaps in between the outer approximations of whole eigenvalues sets and to find outer approximations for individual eigenvalue sets of real or complex interval matrices using their sufficient regularity conditions.

The aim of this paper is to describe two approaches to establish verifiable sufficient regularity conditions of complex interval matrices. In the first approach, a complex interval matrix is mapped to a real block interval matrix and then its sufficient regularity conditions are obtained. In the second approach, a necessary condition for the singularity of a complex interval matrix is derived and used to get its sufficient regularity conditions. As an application, the above derived sufficient regularity conditions are used to investigate the location of the outer approximations of individual eigenvalue sets of complex interval matrices. Two algorithms are proposed and results obtained are compared with those obtained by earlier methods and Monte Carlo simulations. The advantages of these algorithms are that they can detect gaps in between the approximations of the whole eigenvalue sets much better than the existing methods. The second algorithm is very effective compared to the first algorithm from the computational time point of view. Several numerical examples and statistical experiments are worked out to validate and demonstrate the efficacy of our work.

The paper is arranged as follows. Section 1 is the introduction. In Section 2, the preliminaries including concepts, notations, definitions used in the paper is discussed in order to make it self sufficient. The conditions of regularity of real interval matrices are reviewed in Section 3 and with their help, some sufficient regularity conditions for complex interval matrices are developed. Sufficient regularity conditions for complex interval matrices using their necessary singularity conditions are studied in Section 4. In Section 5, the sufficient conditions obtained in Sections 3 and 4 are applied to develop some algorithms for approximating the eigenvalues sets of complex interval matrices. A number of numerical examples and statistical experiments are worked out and results obtained are compared with those obtained by other methods are displayed in graphical form in Section 6. Finally, conclusions are included in Section 7.