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Optimal harvesting of a stochastic delay competitive Lotka–Volterra model with Lévy jumps

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ABSTRACT

This paper systematically investigates the optimal harvesting of a stochastic delay competitive Lotka–Volterra model with Lévy jumps. Under some simple assumptions, the sufficient conditions for extinction and stable in the time average of each species are established. The stability in distribution of this model is proved under our assumptions. Finally, the sufficient and necessary criteria for the existence of optimal harvesting policy are established and the explicit expression of the optimal harvesting effort and the maximum of sustainable yield are also obtained. And some numerical simulations are introduced to demonstrate the theoretical results.

1. Introduction

This work develops the optimal harvesting policies (OHP) for stochastic delay competitive Lotka–Volterra model with Lévy jumps of ecosystems. It is well known that over-harvesting and unreasonable harvesting policies could cause a number of detrimental effects, such as ecological destruction, species extinction and so on. Therefore, the study of OHP and sustainable yields problems in managing natural resources is a significant and important theme in ecology. In recent years, several authors have paid their attention to the field of formulating OHP (see e.g., [1–5]). On the other hand, competition is one of most universal phenomenon in nature. Many researchers have investigated competitive systems and obtained a lot of successful results (see e.g., [6–8]). Hence, it is very significant and meaningful to study the optimal harvesting of a competitive model.

The studies of deterministic models play an important role in mathematical biology, but as noted by Situ [9], "In the practical case a dynamical system will always be disturbed by some stochastic perturbation, one type of which is continuous, and it can be modeled by some stochastic integral with respect to the BM, and the other is of the jump type, which is usually modeled by some stochastic integral with respect to the martingale measure generated by a point process." Some researchers (see e.g., [10–15]) have considered the continuous noises into their population models by supposing that the continuous noises (white noises) mainly affect the growth rate r_i, with $r_i = r_i + \alpha_i \dot{B}_i(t)$, where α_i^2, $i = 1, 2$ denote the intensity of the white noise. $B_i(t)_{t\geq 0}$ are standard independent Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{F}_{t\geqslant 0}, \mathscr{P})$. On the other hand, [16–21] have pointed out that Lévy process can describe the jump type. Liu and Bai [21] considered a stochastic one-prey two-predator model with Lévy jumps, they established the critical values between persistence and extinction for each species and proved the stability in distribution of their model.

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Furthermore, we cannot neglect time delays is important in ecosystem model “since a time delay could cause a stable equilibrium to become unstable and cause the populations to fluctuate” (see e.g., Ruan [22]). Recently, time delays have been used in many population models (see e.g., [23–26]). Qiu and Deng [26] discussed the optimal harvesting problem of a stochastic delay logistic model with Lévy jumps, which is a single-species model. But works on harvesting problems of the stochastic multi-species delay models with Lévy jumps are rare. In this paper, we consider a stochastic delay competitive Lotka–Volterra model with Lévy jumps:

$$\begin{cases} dx_1(t) = x_1(t^-)\Big[r_1 - h_1 - c_{11}x_1(t^-) - c_{12}x_2(t-\tau_1)\Big]dt \\ \qquad\qquad +\alpha_1 x_1(t^-)dB_1(t) + x_1(t^-)\int_{\mathbb{Z}} \gamma_1(\nu)\widetilde{N}(dt,d\nu), \\ dx_2(t) = x_2(t^-)\Big[r_2 - h_2 - c_{21}x_1(t-\tau_2) - c_{22}x_2(t^-)\Big]dt \\ \qquad\qquad +\alpha_2 x_2(t^-)dB_2(t) + x_2(t^-)\int_{\mathbb{Z}} \gamma_2(\nu)\widetilde{N}(dt,d\nu), \end{cases} \tag{1}$$

with initial data

$$x_i(\theta) = \phi_i(\theta), \quad \theta \in [-\tau, 0], \quad \tau = \max\{\tau_1, \tau_2\}, \quad i = 1, 2,$$

where x_1 and x_2 stand for the population size of two species, respectively. $x_i(t^-)$ is the left limit of $x_i(t)$, $i = 1, 2$. $r_i > 0$ is the growth rate of x_i, $i = 1, 2$. $h_i > 0$ represents the harvesting effort of x_i, $i = 1, 2$. $c_{ii} > 0$ is the intraspecific competition coefficients of x_i, i=1,2; c_{ij} ($i \neq j$; i, j=1,2) denotes the interspecific competition rate, respectively. $\tau_i \geq 0$, $i = 1, 2$ are time delays. $\phi_i(\theta) > 0$, i=1,2 are continuous functions defined on $[-\tau, 0]$. $\widetilde{N}(dt, d\nu) = N(dt, d\nu) - \mu(d\nu)dt$, N is a Poisson counting measure, μ is the characteristic measure of N on a measurable subset $\mathbb{Z}$ of $(0, +\infty)$ with $\mu(\mathbb{Z}) < +\infty$. γ_i is the effect of Lévy noises on species i: if $\gamma_i(\nu) > 0$, the jumps represent the increasing of the species; if $\gamma_i(\nu) < 0$, the jumps represent the decreasing of the species. Therefore, It is reasonable to assume that $1 + \gamma_i(\nu) > 0$, $\nu \in \mathbb{Z}$, $i = 1, 2$.

According to our model (1), we will concentrated our research work on the following problems:

(Q1) Which factors influence the expectation of sustainable yield (ESY)?
(Q2) How does the environmental perturbations affect the optimal harvesting policy?

To solve these problems and get the optimal harvesting effort (OHE) $H^* = (h_1^*, h_2^*)$ such that ESY $Y(H) = \lim_{t\to+\infty} \sum_{i=1}^{2} \mathbb{E}(h_i x_i(t))$ is maximum in the case of all species are persistent. First, we establish the sufficient conditions for the extinction and stable in the time average of each species (Section 2). Then we prove the stability in distribution of this model (Section 3). We establish the sufficient and necessary criteria for the existence of the optimal harvesting policy and obtain the explicit expression of OHE and the maximum of ESY (MESY) (Section 4). Finally, we give some numerical simulations to demonstrate our theoretical results (Section 5).

2. Extinction and persistence

For the sake of convenience in the following discussion, we define some notations.

$$\beta_i = h_i + \frac{\alpha_i^2}{2} + \int_{\mathbb{Z}} \Big[\gamma_i(\nu) - \ln(1 + \gamma_i(\nu))\Big]\mu(d\nu), \quad b_i = r_i - \beta_i, \quad i = 1, 2;$$

$$R_+^2 = \{a = (a_1, a_2) \in R^2 \big| a_i > 0, i = 1, 2\}, \quad \Delta = c_{11}c_{22} - c_{21}c_{12};$$

$$\Delta_1 = c_{22}r_1 - c_{12}r_2, \quad \Delta_2 = c_{11}r_2 - c_{21}r_1;$$

$$\widetilde{\Delta}_1 = c_{22}\beta_1 - c_{12}\beta_2, \quad \widetilde{\Delta}_2 = c_{11}\beta_2 - c_{21}\beta_1;$$

$$\langle f(t)\rangle = t^{-1}\int_0^t f(s)ds, \quad \langle f\rangle^* = \limsup_{t\to+\infty} t^{-1}\int_0^t f(s)ds;$$

$$\langle f\rangle_* = \liminf_{t\to+\infty} t^{-1}\int_0^t f(s)ds;$$

Before we state our results, we make some assumptions in the following:

Assumption 1. $\Delta > 0$, $\Delta_i > 0$, $i = 1, 2$. This illustrates that when there are no environment fluctuations (i.e., $\alpha_i = 0$, $\gamma_i(\nu) = 0$) and appropriate harvesting effort, model (1) has a globally stable positive equilibrium (see e.g., Gopalsamy [27]).