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A remark on joint sparse recovery with OMP algorithm under restricted isometry property☆

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ABSTRACT

The theory and algorithms for recovering a sparse representation of multiple measurement vector (MMV) are studied in compressed sensing community. The sparse representation of MMV aims to find the K-row sparse matrix X such that $Y = AX$, where A is a known measurement matrix. In this paper, we show that, if the restricted isometry property (RIP) constant δ_{K+1} of the measurement matrix A satisfies $\delta_{K+1} < \frac{1}{\sqrt{K+1}}$, then all K-row sparse matrices can be recovered exactly via the Orthogonal Matching Pursuit (OMP) algorithm in K iterations based on $Y = AX$. Moreover, a matrix with RIP constant $\delta_{K+1} = \frac{1}{\sqrt{K+0.086}}$ is constructed such that the OMP algorithm fails to recover some K-row sparse matrix X in K iterations. Similar results also hold for K-sparse signals recovery. In addition, our main result further improves the proposed bound $\delta_{K+1} = \frac{1}{\sqrt{K}}$ by Mo and Shen [12] which can not guarantee OMP to exactly recover some K-sparse signals.

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1. Introduction

Compressed sensing has been a very active field of recent research with a wide range of applications, including signal processing [1], medical imaging [2], radar system [3], and image compression [4]. A central goal is to develop fast algorithms that can recover sparse signals from a relatively small number of linear measurements. The *single measurement vector* (SMV) formulation is now standard in sparse approximation and compressed sensing literature. For a signal $x \in R^n$, define $\|x\|_0$ to be the number of nonzero elements of x and the support of x as $supp(x) = \{i : x_i \neq 0\}$ where x_i denotes the ith entry of x. Now we want to recover the original signal x from a linear measurement $y = Ax$, where A is a known $m \times n$ matrix($m \ll n$). Then we need to solve the problem

$$\min \|x\|_0 \quad \text{s.t.} \quad Ax = y, \tag{1.1}$$

where A and y are known.

A natural extension of the problem (1.1) is the *joint sparse recovery* problem, also known as the *multiple measurement vector* (MMV) problem. It aims to identify a common support shared by unknown sparse vectors $x_1, \ldots, x_n$ from the multiple vectors $y_k = Ax_k$, for $k = 1, \ldots, n$, and obtained through a common sensing matrix A. The MMV problem is considered in [5–7],

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where the objective is to minimize the number of rows containing nonzero entries. For the matrix $X = (X_1, X_2, ..., X_l) \in R^{n\times l}$, the row support set of X is

$$supp(X) = \bigcup_{i=1}^{l} supp(X_i),$$

where X_i represents the ith column of matrix X. Given a multiple-measurement vector $Y \in R^{m\times l}$ and a measurement matrix $A \in R^{m\times n}$, where the columns of A have been normalized, the MMV problem can be formulated as

$$\min |supp(X)| \quad \text{s.t.} \quad AX = Y, \tag{1.2}$$

where $|supp(X)|$ is defined as the number of nonzero rows of the matrix X. The matrix X is called *K-row sparse* if $|supp(X)| \le K$. A sparse representation means that matrix X (or a vector, if one has an SMV with $l = 1$) has a small number of rows that contain nonzero entries. The Orthogonal Matching Pursuit (OMP) algorithm can find the sparsest solution X of problem (1.2) when some conditions are satisfied. In this paper, Λ is used to denote the row support set of the sparsest solution X of problem (1.2).

The following recovery guarantees are based on the restricted isometry property (RIP) introduced by Candés and Tao [8]. The standard K-order RIP constant of A is the smallest nonnegative real number δ_K such that

$$(1 - \delta_K)\|x\|_2^2 \le \|Ax\|_2^2 \le (1 + \delta_K)\|x\|_2^2$$

for all K sparse vector x (i. e., $|supp(x)| \le K$).

In the last few years, numerous algorithms have been proposed and studied for solving the SMV problem. Another way to obtain a sparse representation is through a greedy algorithm, e.g., OMP [5,6]. It has been proved by Donoho et al. [1] and Tropp [9] independently that under certain conditions, the OMP can find the sparsest representation of the signal. Considering the conditions in terms of RIP constant for OMP to exactly recover any K-sparse signal in K iterations, Davenport and Wakin [10] have proven that $\delta_{K+1} < \frac{1}{3\sqrt{K}}$ is sufficient. Liu and Temlyakov [11] have improved the condition to $\delta_{K+1} < \frac{1}{(\sqrt{2}+1)\sqrt{K}}$. Later, Mo and Shen [12] have shown that $\delta_{K+1} < \frac{1}{\sqrt{K+1}}$ is near-optimal, and also constructed a matrix with RIP constant $\delta_{K+1} = \frac{1}{\sqrt{K}}$ such that the OMP can not recover some K-sparse signal x in K iterations. Recently, Zhao et al. [13] further show that $\delta_{K+1} < \frac{1}{\sqrt{K+1/13}}$ still does not guarantee that some K sparse signals be recovered exactly. However, Ding et al. [14] have shown that the condition $\delta_{K+1} < \frac{1}{2\sqrt{K+1}}$ is sufficient for the exact recovery of a K-row sparse matrix X via the OMP algorithm to solve the MMV problem in the noiseless case.

In this paper, we propose an improved condition to guarantee the exact recovery of the OMP algorithm for MMV problem. Our main conclusion states that if the RIP constant of the sensing matrix A satisfies $\delta_{K+1} < \frac{1}{\sqrt{K+1}}$, then the OMP algorithm can exactly recover any K-row sparse matrix from the measurements $Y = AX$. Moreover, we also construct a matrix with RIP constant $\delta_{K+1} = \frac{1}{\sqrt{K+0.086}}$ such that the OMP algorithm fails to recover some K-row sparse matrix. Similar results also hold for K-sparse signals recovery.

The rest of the paper is organized as follows. In Section 2, we shall introduce some notations. In Section 3, we shall give a simple observation of the OMP algorithm to solve the problem (1.2), and a sufficient condition for the OMP algorithm to exactly recover any K-row sparse matrix is established. we provide an example to show that there exists a matrix with RIP constant $\delta_{K+1} = \frac{1}{\sqrt{K+0.086}}$ for which OMP algorithm fails to recover the K-sparse signal. Similar results also hold for K-sparse signals recovery. Finally, Section 4 concludes the paper and gives some discussion on related work.

2. Notations

In this section, we introduce some basic notations that will be used throughout the paper.

For matrices X and Y in $R^{m\times n}$, we define the inner product by $\langle X, Y\rangle = \mathrm{tr}(X^TY) = \sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}Y_{ij}$, where X^T denotes the transpose of X. The norm induced by this inner product is the Frobenius (or Hilbert–Schmidt) norm $\|\cdot\|_F$, and $\|X\|_\infty$ is the maximum absolute row sum of the matrix X. We use $X_{(i)}$ to denote the ith row of X.

Let $\Lambda \subset \{1, 2, ..., n\}$ and $\Lambda^c = \{1, 2, ..., n\}\backslash\Lambda$ be index sets. For $A \in R^{m\times n}$, A_Λ is $m \times |\Lambda|$ matrix obtained by selecting the columns of A indexed by Λ. We define the range space of A by $\mathcal{R}(A) = \{y : y = Ax,\ \ for\ x \in R^n\}$. Let $A\dagger$ be the Moore–Penrose pseudoinverse of A. If A is a full column rank matrix, then $A^\dagger = (A^TA)^{-1}A^T$. Let $P_A = AA^\dagger$ be the orthogonal projection operator onto $\mathcal{R}(A)$, then $(I - P_A)$ is the orthogonal projection operator onto the orthogonal complement of $\mathcal{R}(A)$.

3. Main results

In this section we present a detailed description of the Orthogonal Matching Pursuit (OMP) algorithm. We assume that the columns of A are normalized, i.e., $\|A_i\|_2 = 1$ for $i = 1, 2, \dots, n$. The OMP algorithm to solve problem (1.2) in [5,14] can be stated as follows.

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