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Robust consensus control for a class of multi-agent systems via distributed PID algorithm and weighted edge dynamics

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ABSTRACT

This paper studies the robust proportional-integral-derivative (PID) consensus control for a class of linear multi-agent systems (MASs) with external disturbances. Different from the existing results, both the consensus analysis and the transient performance characteristics for high-order linear MASs are considered. Based on a factorization of Laplacian matrix, the initial MAS is firstly transformed into the so-called weighted edge dynamics, and then a design equivalence between the proposed PID consensus controller and the corresponding stabilizing controller for such weighted edge dynamics is presented via some graph theory results. Furthermore, by combining Lyapunov theory and Barbalat's Lemma, it is proved that both the stabilization of weighted edge dynamics and the consensus of MAS can be guaranteed even in the presence of external disturbances. In particular, some relationships between the transient time performance of weighted edge dynamics and the PID design parameters are given. Finally, some numerical examples on LC oscillator network are provided to illustrate the validity of theoretical results.

1. Introduction

The control of networked systems has received much attention over the past few years due to its widely applications in many fields, such as electrical circuit systems [1,2], neural networks [3–5], communication networks [6,7], chaotic systems [8] and autonomous mobile robots [9]. Especially, the consensus (or synchronization) control for multi-agent systems (MASs) is intensively studied by amounts of researchers. The so-called consensus means that all agents reach an agreement on certain quantities via an appropriate distributed control law [11].

In general, there are two types of consensus problems that are often considered in practice, namely the leaderless consensus and the leader-following consensus. As to the leaderless one, the main objective is to design a distributed control law such that the states of all agents converge to an unprescribed common value, while the leader-following one is to make all agents track the leader's trajectory [10,11]. Since the pioneering works [12–14], a great number of leaderless and leader-following consensus approaches are developed in the literature. For example, a coupled-group consensus problem for MASs with both fixed and switched topologies is considered in [15]. Qin et al. [16] investigate the leaderless consensus control for discrete-time MASs under time-varying dynamic topology. In [17], a neural network-based approximation technique is used to solve the leaderless consensus problem for nonlinear MASs with state time-delay. On the other hand, some sufficient conditions for the stabilization of switching nonlinear systems are given in [18], and this result is used to solve the

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leader-following consensus problem for MASs. In addition, some interesting synchronization control approaches are given in [19–21] for chaotic systems.

Although these efforts, all the aforementioned methods assume that the system model is operated in ideal environment. In many practical applications, the networked control systems may often encounter random abrupt variations in their parameters or structures [22,23]. Especially, the external disturbances as well as the model imperfections may often appear in the plants. Thus the robust control plays an important role in the consensus of MASs. Recently, the well-known proportional-integral (PI) control strategy is commonly applied to attenuate unknown disturbances [24], and some effective distributed PI consensus algorithms are presented. For instance, a distributed proportional-integral controller for networked dynamical systems is designed in [24] to successfully attenuate the constant disturbances in the communication network. [25] further investigates the PI consensus controller design for high-order linear MASs, and in [26], the transient time performance characteristics of the integral action-based consensus approach are provided via $\mathcal{L}$ stability analysis.

On the other hand, the derivative action-based consensus protocol also attracts much attention from researchers since it can significantly increase the distributed control freedom [27], based on which some proportional-integral-derivative (PID) consensus methods are provided. In [28], a transverse dynamics of MASs model is derived via a block decomposition of Laplacian matrix, based on which the PID consensus analysis is conducted. In [29], a PI^n-type containment control approach for MASs is developed, where I^n denotes the corresponding algorithm includes up to the nth order integral terms, and this result further extends to the case with $PI^{n-m}D^m$-type algorithm. Nevertheless, the system models considered in [28,29] are only belong to the first-order linear dynamics or integrator dynamics. How to address the PID consensus problem for high-order linear MASs has not been fully investigated up to now. Moreover, it is also an interesting issue to give some relationships between the PID design parameters and the transient performance characteristics of MASs. These reasons motivate the current investigations.

Recently, some interesting consensus approaches are developed in [30,31] based on the so-called “edge agreement protocol”, which provide a new perspective on the consensus algorithm by making a transformation from node dynamics to edge dynamics. Inspired by these results, a novel distributed PID consensus control scheme for high-order linear MASs is presented in this paper. A transformed system named as weighted edge dynamics is firstly introduced, and then a design equivalence between the distributed PID controller for initial MASs and the stabilizing controller for weighted edge dynamics is given via some graph theory results. Simultaneously, it is proved that the consensus for MASs can be achieved under the proposed edge-based PID algorithm even in the presence of external disturbances. Furthermore, some descriptions about the transient time performance for weighted edge dynamics are presented via $\mathcal{L}_2$ and $\mathcal{L}_\infty$ norms. Compared with the existing results, the main contributions and advantages of our work consist of:

- Different from the existing PID consensus methods [28,29], where only first-order linear dynamics or integrator dynamics are considered, this paper focuses on solving the consensus problem for high-order linear MASs.
- A novel edge-based PID control approach is given in this paper. Compared with [28], this method is independent on the block decomposition of Laplacian matrix, which simplifies the process of consensus analysis.
- This paper provides some relationships between the PID design parameters and the transient performance characteristics of MASs, while the works [24,25,27,29] are not considered.
- In fact, the so-called weighted edge dynamics has been presented in our previous work [32]. However, this result only gives a proportional consensus method for MASs. In comparison, this paper extends our previous result to the PID consensus control.

The paper is organized as follows: The problem formulation is given in Section 2. In Section 3, the weighted edge dynamics is introduced, and the consensus analysis of the considered MAS is conducted. In Section 4, we further discuss the transient time performance characteristics of the proposed PID consensus method. The numerical examples are presented in Section 5. Finally, the conclusions are derived in Section 6.

2. Problem formulation

2.1. Preliminaries

In this paper, $\mathbb{R}$, $\mathbb{C}_-^0$ and $\mathbb{Z}_+$ denote the real set, complex with non-positive real part set and positive integer set, respectively. For any matrix $A \in \mathbb{R}^{n\times m}$, A^T represents its transpose. If A is a square matrix, $\sigma(A)$ denotes the eigenvalue set of A. $\mathbf{1}_N$ denotes the $N\times 1$ vector with all elements equal to 1, and I_N denotes the $N\times N$ identity matrix. $\mathbf{0}$ represents any matrix where the elements are all zero. $\otimes$ represents the Kronecker product. For a vector $v \in \mathbb{R}^n$, $\|v\|$ and $\|v\|_\infty$ denote the Euclidian norm and ∞ norm, respectively. For a signal $x(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$, the $\mathcal{L}_2$ norm and $\mathcal{L}_\infty$ norm [26] are respectively defined as

$$\|x(t)\|_{\mathcal{L}_2} \triangleq \sqrt{\int_0^\infty \sum_{i=1}^n x_i^2(t)dt} \tag{1}$$

$$\|x(t)\|_{\mathcal{L}_\infty} \triangleq \max_{1\le i\le n}\left(\sup_{t\ge 0}|x_i(t)|\right) \tag{2}$$