Applied Mathematics and Computation 316 (2018) 115–137

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Dynamic analysis of a hybrid bioeconomic plankton system with double time delays and stochastic fluctuations

Chao Liu[a,b,c,*], Longfei Yu[a,c], Qingling Zhang[a,c], Yuanke Li[d]

[a] Institute of Systems Science, Northeastern University, Shenyang, China
[b] Department of Applied Mathematics, Hong Kong Polytechnic University, Kowloon, Hong Kong
[c] State Key Laboratory of Integrated Automation of Process Industry, Shenyang, China
[d] Faculty of Engineering, University of New South Wales, Sydney, Australia

ARTICLE INFO

Keywords:
Bioeconomic plankton system
Singularity induced bifurcation
Stochastic stability
Stochastic Hopf bifurcation

ABSTRACT

In this paper, we establish a double delayed hybrid bioeconomic plankton system with stochastic fluctuations and commercial harvesting on zooplankton, where maturation delay for toxin producing phytoplankton and gestation delay for zooplankton are considered. Stochastic fluctuations are incorporated into the proposed system in form of Gaussian white noises to depict stochastic environmental factors in plankton system. For deterministic system without double time delays, existence of singularity induced bifurcation is studied due to variation of economic interest of commercial harvesting, and state feedback controllers are designed to eliminate singularity induced bifurcation. For deterministic system with double time delays, positivity and uniform persistence of solutions are studied, and some sufficient conditions associated with asymptotic stability of interior equilibrium are investigated. For stochastic system without double time delays, stochastic stability and existence of stochastic Hopf bifurcation are discussed based on singular boundary theory of diffusion process and invariant measure theory. For stochastic system with double time delays, existence and uniqueness of global positive solution are investigated, and asymptotic behaviors of the interior equilibrium are studied by constructing appropriate Lyapunov functions. Numerical simulations are carried out to validate theoretical analysis.

1. Introduction

In marine ecosystem, some specific phytoplankton population release toxin substances and reduce predation pressure from zooplankton population in plankton system [2,6,17]. Furthermore, some practical observations from real marine ecosystem reveal that liberation process of toxin substances is not instantaneous but delayed by certain maturation duration of toxin producing phytoplankton in marine ecosystem [2,3,6,11]. In recent years, some mathematical models composed of delayed differential equations have been utilized to discuss complex dynamics and stability analysis of plankton system with toxin producing phytoplankton [22–25,27,28,31,32,38,46]. The oscillatory situations and Hopf bifurcation phenomenon around interior equilibrium due to variation of time delay are discussed in [22–24,32,38]. The destabilizing effect of time delay and some sufficient conditions associated with persistence of the interior equilibrium are studied in [23,25,27,28,46]. Based on field-collected samples, a phytoplankton zooplankton system with toxin liberation delay $\tau > 0$ is established in

* Corresponding author at: Institute of Systems Science, Northeastern University, Shenyang, China.
E-mail address: liuchao@mail.neu.edu.cn (C. Liu).

http://dx.doi.org/10.1016/j.amc.2017.08.019

[31] to reflect population dynamics depending on maturation delay of toxin producing phytoplankton, which is as follows,

$$\begin{cases} \dot{P}(t) = rP(t)\left(1 - \frac{P(t)}{K}\right) - \frac{\beta P(t)Z(t)}{\alpha+P(t)}, \\ \dot{Z}(t) = \frac{\beta_1 P(t)Z(t)}{\alpha+P(t)} - dZ(t) - \rho P(t-\tau)Z(t), \end{cases} \tag{1}$$

where $P(t)$ and $Z(t)$ denotes toxin producing phytoplankton population density and zooplankton population density, respectively. r represents intrinsic growth rate of toxin producing phytoplankton, d stands for natural decay rate of zooplankton, K represents environmental carrying capacity of toxin producing phytoplankton. β is supreme consumption rate of zooplankton, β_1 denotes biomass conversion rate such that $0 < \beta_1 < \beta$. ρ represents toxin substances liberation rate. Supposing that rate of toxin substance liberation is less than rate of biomass consumed by zooplankton, it is assumed that $\rho < \beta_1$. Complex dynamical behavior and local stability analysis around the interior equilibrium are discussed in [31].

It is well known that some zooplankton such as aretes, jellyfish and krill may be commercially exploited from the view points of human medical and physical need [13,16,18,35,40]. In recent years, dynamic effects of harvest effort on zooplankton population have drawn a great deal of attentions [26,30,36,39], which show that an appropriate commercial harvest effort may also guarantee sustainable development of harvested zooplankton population in marine system. From the perspective of economy theory viewpoint, it should be noted that commercial harvesting fluctuates due to variation of economic interest of commercial harvesting [13]. However, little research work has been made on dynamic effects of economic interest of commercial harvesting on population dynamics. Gordon [1] proposed common-resource properties theory from an economic viewpoint, which is as follows:

$$\text{Net Economic Revenue} = \text{Total Revenue (TR)} - \text{Total Cost (TC)}. \tag{2}$$

Furthermore, uncertainty in population growth is usually regarded as one of important themes in plankton system. Especially, some stochastic fluctuations are considered to be dynamic effect of environmental stochasticity, which refers to temperature and nutrient density within surrounding marine environment show oscillation around specific average state in plankton system [12]. Hence, population dynamics of plankton system can not be totally depicted and investigated by utilizing deterministic mathematical model [12]. Recently, Gaussian white noises are proved to efficiently depict rapidly fluctuating phenomena arising from marine ecosystem in the real world [5,19].

Based on the above analysis, some hypotheses are proposed as follows:

(H1) In this paper, it is assumed that zooplankton is commercially harvested, $E(t)$ represents commercial harvesting on zooplankton, w represents unit price of harvested zooplankton, c and v denotes cost and economic interest of commercial harvesting, respectively. Hence, it follows from Eq. (2) that

$$\begin{cases} \text{TR} = wE(t)Z(t), \quad \text{TC} = cE(t), \\ v = E(t)(wZ(t) - c). \end{cases} \tag{3}$$

(H2) In this paper, it is assumed that the liberation of toxin substances is not instantaneous but mediated by some time lag which is required for toxin producing phytoplankton maturation. The reproduction of zooplankton after predating toxin producing phytoplankton is not instantaneous but will be mediated by some time lag required for zooplankton gestation. We will extend the work in [31] by incorporating discrete time delay for gestation of zooplankton population into system (1). $\tau_1 \geq 0$ denotes gestation delay for zooplankton, and $\tau_2 \geq 0$ represents maturation delay for toxin producing phytoplankton.

(H3) In this paper, population growth of toxin producing phytoplankton and zooplankton affected by environmental stochasticity are all assumed to be stochastic process rather than a deterministic process. Gaussian white noises will be introduced into the proposed system to depict stochastic environmental factors in plankton system.

By using hypotheses (H1)-(H3), a double delayed bioeconomic phytoplankton zooplankton system with stochastic fluctuations is as follows,

$$\begin{cases} \dot{P}(t) = rP(t)\left(1 - \frac{P(t)}{K}\right) - \frac{\beta P(t)Z(t)}{\alpha+P(t)} + \omega_{11}P(t)\xi_1(t) + \omega_{12}\xi_2(t), \\ \dot{Z}(t) = \frac{\beta_1 P(t-\tau_1)Z(t-\tau_1)}{\alpha+P(t-\tau_1)} - dZ(t) - \rho P(t-\tau_2)Z(t) - E(t)Z(t) + \omega_{21}Z(t)\xi_1(t) + \omega_{22}\xi_2(t), \\ 0 = E(t)(wZ(t) - c) - v, \end{cases} \tag{4}$$

where ω_{jk} $(j, k = 1, 2)$ are nonnegative constants, $\xi_1(t)$ and $\xi_2(t)$ represents multiplicative stochastic excitation and external stochastic excitation related to marine environment, respectively. $\xi_j(t)$ $(j = 1, 2)$ denotes independent Gaussian white noise such that $\mathbb{E}[\xi_j(t)] = 0$ $(j = 1, 2)$. For discrete time $t_1 \neq t_2$, $\mathbb{E}[\xi_j(t_1)\xi_j(t_2)] = \delta(t_2 - t_1)$ $(j = 1, 2)$, and $\mathbb{E}[\xi_j(t_1)\xi_k(t_2)] = 0$ $(j, k = 1, 2, j \neq k)$, where δ denotes Dirac delta function.

Other parameters share the same interpretations mentioned in (1) and hypotheses (H1)–(H3). The initial conditions for system (4) are as follows,

$$P(\theta) > 0, Z(\theta) > 0, E(0) > 0, \theta \in [-\tau_m, 0], \tag{5}$$

where $\tau_m = \max\{\tau_1, \tau_2\}$.