Applied Mathematics and Computation 316 (2018) 174–185

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# Co-evolution of cooperation and limited resources on interdependent networks

Chao Luo[a,b,*], Xingyuan Wang[c], Yuanjie Zheng[a,d,e]

[a] School of Information Science and Engineering, Shandong Normal University, Jinan 250014, China
[b] Institute of Data Science and Technology, Shandong Normal University, Jinan 250014, China
[c] Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian 116024, China
[d] Institute of Life Sciences at Shandong Normal University, Jinan 250014, China
[e] Key Laboratory of Intelligent Information Processing at Shandong Normal University, Jinan 250014, China

ARTICLE INFO

*Keywords:*
Interdependent networks
Co-evolution
Resource allocation
Evolutionary games

ABSTRACT

In this article, based on interdependent networks, cooperation in spatial prisoner's dilemma game (PDG) with coevolving resources is studied. By means of a strategy-independent rule, limited resources can be continually re-allocated among different players in the same network or across different networks. The coevolution of dynamics is discussed respectively for two cases: game circumstances on two coupled networks are identical or asymmetry. In the first case, we obtain that the involvement of resources can be significantly beneficial for cooperative behaviors, and the heterogeneous distribution of resources can positively enhance interdependent network reciprocity. Furthermore, an optimal value ($\rho \approx 0.8$) of the interdependent strength exists for cooperation, which is obviously larger than the previous results ($\rho \approx 0.5$) without coevolving resources. Besides, we also find resources follow the power law distribution, where cooperators and interconnected players tend to obtain more resources. In the second case, we mainly focus on the flow of resources between networks as well as the intertwined effect of the interdependent strength and resources distribution on cooperation. Microscopic dynamical properties of the flow of resources jointly caused by various factors have been discussed. In certain conditions, instead of promoting the cooperation, the influx of resources could even be positive for defection.



## 1. Introduction

Game Theory, generally, can be defined as a study based on highly abstract models expressing the conflict / cooperation among decision makers, and provides a common mathematical approach for the analysis of agents involved in pair-wise or group interaction where their decisions affecting the interests of each other [1]. In 1973, inspired by the study of ecological evolution and classical game theory, Smith etc. proposed the concept of Evolutionary Stable Strategy (ESS), which was normally considered as the symbol of the emergence of the evolutionary game theory [2]. In the biological system, according to the principle of survival of the fittest, adaptable individuals would replace the low adaptability of individuals. Hence, based on statistical physics [3], to understand the emergence of altruistic behaviors in the context of Darwinian evolution, rational

* Corresponding author at: School of Information Science and Engineering, Shandong Normal University, Jinan 250014, China.
*E-mail address:* cluo79@gmail.com (C. Luo).

http://dx.doi.org/10.1016/j.amc.2017.08.018
0096-3003/

players in the classical games are substituted by limited rational individuals with mutation and reproduction. Since then, evolutionary games have gradually developed into a science of problem solving, and be widely implemented in biology, economics, human ethology, sociology, etc. [4–7].

In recent years, complex networks theory provides a new view and method for studying the system complexity. Based on various structured models, studies on epidemic and sentiment spreading [8,9], evolutionary computation [10–14], control [15–17], etc., have been developed rapidly. It is an important subject for evolutionary game theory that how the structured population exerts an influence on evolutionary dynamics. In earlier research, evolutionary games were usually assumed to be carried out in a well-mixed population [18]. However, in reality, individuals in a group are not necessarily fully coupled and also have different degrees of tightness with each other [19]. In 1992, Nowak and May investigated the prisoner's dilemma game based on square lattice [20]. Different from the case in a well-mixed population where a homogenous defection inevitably unwinds, cooperative behaviors could survive by means of network reciprocity on lattice, which verifies the effect of network structure on evolutionary dynamics. Since then, evolutionary games based on different structured models, such as regular lattice [21], small world network [22], scale free network [23], etc., have been widely discussed. Meanwhile, compared with the classical single-layer complex network, interdependent networks [24] can be viewed as a more general system composed of a plurality of parallel interacting networks. In the past few years, interdependent networks have attracted wide attention of the researchers, and numerous of results, including studies on network topology and dynamics [25], the detection of community structure [26], transmission dynamics [27,28], etc., have been proposed. Notably, interdependent networks as a kind of structured population are also implemented into the investigation of evolutionary games. Wang et al. [29] found the emergence of interdependent network reciprocity and verified it could promote the fraction of cooperation on interdependent networks; Szolnoki and Perk [30] proposed sharing information between coupled networks could be beneficial for cooperative behaviors; Gómez-Gardeñes et al. [31] presented the resilience of cooperative behaviors for extremely large values of the temptation to defect could be enhanced by the multiplex structure. Besides, from different angles, various studies based on interdependent networks, for instance, self-organization [32], spontaneous symmetry breaking [33], spreading of strategies [34], the memory ability [35], an optimal probabilistic interconnection [36], etc., have also been implemented.

Various parts inside things are interrelated, i.e. evolution without isolation [37]. In recent years, special concerns have been devoted to co-evolution of evolutionary games [38], under certain pre-defined rules, where game strategies co-evolve with other variable factors, such as network topology [39] and time scale [40]. For instance, in [41], the coevolution of quantum and classical strategies on weighted and directed random networks was studied, where network structure was subject to evolution by agents break and rewire their links with the aim of maximizing payoffs; in Ref. [42], evolutionary games were carried out accompanied by the evolution of the teaching activity of players, where strategy adoption was determined by the difference of payoff and a teaching activity characterizing the donor's capability to enforce its strategy on the opponent; in Ref. [43], researchers found high levels of cooperation can be attained when networks are growing, but the same structure of network does not promote cooperation as a static network. Furthermore, based on interdependent networks, some studies on co-evolution of evolutionary games were also proposed [44,45].

In this article, PDG is implemented on interdependent networks, where each player on networks is assigned with certain amount of resources. Here, resources refer to some valuable factors, such as wealth, water, energy, etc. In [46], the coevolving resources on a single square lattice were discussed, where the evolutionary game was implemented without influences from outside the population. In [47], common resources are involved in the collective-risk social dilemma game, which is as a measure of whether the collective target is achieved or not. Researchers found that cooperation can be sustained only if the common resources are preserved but never excessively abound. Considering the significant impact derived from the structure of population, interdependent networks as a more general model are utilized, where resources can be continually reallocated among players on the same network and also can flow across different networks along with the evolution of game strategies. Mainly, we are interested in the intertwined effect of the interdependence and resources distribution on cooperation and studies are investigated as follows. Firstly, based on interdependent networks, the effect of coevolving resources on cooperation is discussed. As shown in previous studies [29,32], the interdependent network reciprocity existing in interdependent networks can boost cooperative behaviors. We focus on whether the spontaneous emergence of resource distribution on interdependent networks can further have a positive effect on cooperation. Secondly, the distribution of resources is analyzed when system approaches equilibrium. Respective from microscopic and macroscopic views, the relationship between resources distribution and the cooperation level is studied. Finally, with different game circumstances on coupled networks, the co-evolutionary dynamics would be investigated. Especially, how resources flow across different layers and its effect on dynamics are discussed.

The rest of this paper is organized as follows. In Section 2, the proposed evolutionary game with resources on interdependent networks is given. Next, the simulation studies are reported and their implications are discussed. Finally, a concluding remark is given.

## 2. Model

In this article, evolutionary game is implemented on interdependent networks, which is composed by two coupled square lattices of size $L \times L$ with periodic boundary conditions. For the convenience of discussion, two lattices are called as network $A$ and $B$. Each participant plays game with its four nearest local neighbors connected by internal links during the