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Finite-time stability and stabilization of switched nonlinear systems with asynchronous switching

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ABSTRACT

This paper is concerned with the problem of finite-time stability and stabilization for switched nonlinear systems with asynchronous switching. Firstly, when not all subsystems are finite-time stable (FTS), we propose a finite-time stability criteria to show that if the constraint condition between the settling time and the average dwell time is satisfied, finite-time stability of the system is guaranteed. Then we extend the result to the case which Lipschitzian perturbations exist in the subsystems. When asynchronous switching is considered and not all closed-loop subsystems are finite-time stabilizable, we recur to the generalized inverse of matrices to design a state feedback controller to stabilize the original system. Finally, an example of two container liquid-level system is provided to illustrate the effectiveness of developed result.

1. Introduction

Switched nonlinear systems are a class of typical nonlinear hybrid systems, and have the most basic composition characteristics of them such as nonlinear hybrid dynamical subsystems and a logical rule orchestrating switching among these subsystems. Many practical systems, which are inherently multi-model and nonlinear, can be modeled by switched nonlinear systems, such as multi-agent systems [1], network control systems [2] and so on. In recent years, stability analysis and stabilization of switched nonlinear systems have gained considerable interest, for example, controller design under arbitrary or designed switchings [3,4] and incremental (Q, S, R)- dissipativity stability under state-dependent switching law [5].

For above mentioned systems, considerable attention has been paid to the asymptotic stability and the exponential stability. It has been proved that some inherently nonlinear systems, which cannot be stabilized by any smooth feedback control method, may be stabilized by using finite-time control methods [6,7]. Finite-time control is always playing an important role in stabilization of nonlinear systems. Finite-time stability implies Lyapunov stability and convergence of system trajectories to an equilibrium state in finite time. It was shown in [8] that finite-time stable systems might enjoy not only faster convergence but also better robustness and disturbance rejection properties. Some finite-time stability results have been reported for hybrid systems [9,10]. Recently, backstepping design method has been extensively used in finite-time stabilization. For example, an adaptive fuzzy finite-time control scheme is proposed for switched nonlinear systems with unknown control coefficients and nonlinearities via backstepping methodology in [11]. Cai and Xiang [12] investigates the adaptive finite-time stabilization of switched nonlinear systems with unknown nonlinear terms using neural networks. A common Lyapunov

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function and backstepping design method are used to investigate the finite-time stabilization of switched stochastic nonlinear systems under arbitrary switching [13]. In [14], a common coordinate transformation of all subsystems is exploited to design the controller to guarantee global finite-time stabilization of the closed-loop non-smooth switched system. Unfortunately, these works all used a common Lyapunov function to study finite-time convergence of the system, thus leading to the larger conservatism. The essential reason why a common Lyapunov function is used instead of multiply Lyapunov functions is that multiply Lyapunov functions will lead to different coordinate transformations for different subsystems when using backstepping strategy. This means that when designing switched controller for each subsystem, we have to construct Lyapunov functions under different coordinate transformation which is extremely difficult. By a common Lyapunov function, we can have a common coordinate transformation if all subsystems share a common stabilizing function during each step of backstepping, which makes the stabilization problem become solvable but conservative. In this paper, we attempt to develop multiply Lyapunov functions method to reduce the conservativeness of the controller design, and in the process of applying it, we find that a finite-time stabilizable condition can be derived by means of a tool of the generalized inverse of matrices to avoid difficulty in different coordinate transformation of backstepping method, which is less conservative.

Recently, asynchronously switched stabilization of switched systems has been a hot topic in the control area [15–20] because of its practical interest and theoretical significance. The related results have been extended to the case of nonlinear subsystems [21–24]. However, the main drawback of the approach applied in such systems is that the nonlinearities only exist in the perturbation and linearization approach is always used to solve nonlinear control problem. As we know, only considering nonlinear perturbation, those essential nonlinear systems are hard to be controlled by linearization approach. For the case without asynchronous switching, many effective nonlinear analysis methods can be applied to study the stability and stabilization of systems. A necessary and sufficient condition for quadratic stability of switched systems is derived by using Karush–Kuhn–Tucker condition for nonlinear programming problems in [25]. By multiple Lyapunov functions, a kind of new methodology is developed which is the integrated synthesis of “lower-level” bounded nonlinear feedback controllers together with “upper-level” switching laws [26]. Backstepping design method is used to globally stabilize switched nonlinear systems in lower triangular form under arbitrary switching in [3]. The implications of a Lie-algebraic condition are discussed to guarantee the uniformly exponential stability for switched nonlinear systems by Liberzon et al. [27]. However, the mixture of the essential nonlinearity and asynchronous switching gives rise to the difficulty in the control design because existing results usually require the stringent correspondence between the constructed control-Lyapunov functions and nonlinear subsystems. The designed controller for the previous mode may not stabilize the current mode, thus possibly leading to an unstable system. Therefore, it will be academically challenging to develop nonlinear controller design for switched nonlinear systems with asynchronous switching. Considering the effectiveness of finite-time control for nonlinear system, it is worth developing nonlinear control strategy with finite-time stability to solve the problem of asynchronous switching, which is not only theoretically significant, but also important from an application point of view. To the best of our knowledge, this issue has not been addressed in the existing literature, which motivates the present study.

In this paper, we discuss the finite-time stability and stabilization of switched nonlinear systems with asynchronous switching by using the average dwell time method. Based on multiply Lyapunov functions, a finite-time stability criteria relying on the settling time and average dwell time is derived, and then finite-time asynchronously switched control and adaptive switching law are designed with the help of the generalized inverse of matrices. Compared with the previous works, this paper mainly has the following three contributions: (i) This paper considers the finite-time stability of switched nonlinear systems, which are more general hybrid nonlinear systems having just caused a little research up to now. The complexity introduced by asynchronous switching and nonlinear characteristic makes it difficult to check the finite-time stability property but worthwhile in the sense that it may bring much attention to this field. (ii) The finite-time stability mentioned in this paper combines finite-time convergence with Lyapunov stability. Differently from existing works that only require the state of the system is bounded within finite time [28,29], this paper provides an efficient stabilization method, which makes the origin of the system converge to zero within finite time. Particularly, the introduction of the generalized inverse of matrices plays a critical role in constructing switched nonlinear controllers, and the dimensions of control input and state variables in controllable part of systems are allowed to be different, which covers more general cases. (iii) Most of existing works studied finite-time stability of switched nonlinear system with finite-time convergence by using a common Lyapunov function, which leads to conservatism of controller design. In this paper, the construction of switched controller relies on multiply Lyapunov functions, and the relationship between the settling time and the average dwell time is also revealed, which makes finite-time stabilization more realizable.

Notations. Throughout this paper, I is an identity matrix with appropriate dimension. $\mathbb{Z}$ denotes a non-negative integer set. $\mathbb{Z}^+$ is a positive integer set. $\mathbb{R}^+$ is a positive real number set. $\mathbb{Z}^+_{odd}$ and $\mathbb{Z}^+_{even}$ are positive odd number set and even number set. $\max\{\bullet\}$ and $\min\{\bullet\}$ refer to the maximum and minimum, respectively. A^T denotes the transpose of matrix A. B^+ is the generalized inverse of matrix B. $\mathbb{R}^n$ is the n-dimensional real Euclidean space. $\mathbb{R}^{m\times n}$ is the set of all real $m \times n$ matrices. $\|\bullet\|$ is the 2-norm on $\mathbb{R}^n$. $\mathcal{D}$ is an open neighborhood on $\mathbb{R}^n$. $\mathcal{N}$ and $\mathcal{V}$ are an open neighborhood on $\mathcal{D}$.