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Stability and convergence of second order time discrete projection method for the linearized Oldroyd model☆

Jing Zhao[a,*], Tong Zhang[b,c], Yanxia Qian[b]

[a] School of Civil Engineering, Henan Polytechnic University, Jiaozuo 454003, PR China
[b] School of Mathematics & Information Science, Henan Polytechnic University, Jiaozuo 454003, PR China
[c] Departamento de Matemática, Universidade Federal do Paraná, Centro Politécnico, Curitiba 81531-990, Brazil

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ABSTRACT

In this paper, we consider the second order time discrete projection method for the linearized Oldroyd model based on the time iterative discrete scheme. By the projection method, the original problem is decoupled into two linear subproblems, and each subproblem can be solved easily. Unconditional stability and the corresponding convergence results of the numerical solutions are derived. Our main results are that the convergence orders in time for the velocity in L^2-norm is second order and for the pressure in H^1-norm is first order. Finally, some numerical examples are provided to verify the performances of the developed numerical method.

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1. Introduction

For the incompressible flow problem, the main difficulties are the coupling between the velocity and pressure, the incompressible conditions and the nonlinearity term. The projection method is an efficient numerical scheme to treat the multi-variables coupled problem. The most attractive feature of the projection method is that one can decouple the original problem into a sequence of linearized subproblems with different variables, each subproblem can be solved easily and the computational scale is also reduced. Due to the high efficiency of the projection method, much attention has been attracted, for example, we can refer to Chorin [4] and Temam [22] for the ground breaking works of the projection method, [1,5,16–20] for the incompressible flow problem. Due to the efficiency of the projection method, in this paper we consider the second order time discrete projection method for the linearized Oldroyd model:

$$\begin{cases} u_t - \nu\Delta u + (u\cdot\nabla)u + \nabla p - \lambda\int_0^t e^{-\delta(t-s)}\Delta u ds = f, & (x,t)\in\Omega\times(0,T], \\ \text{div } u(x,t) = 0, & (x,t)\in\Omega\times(0,T], \\ u(x,0) = u_0(x), & (x,t)\in\Omega\times\{0\}, \\ u(x,t) = 0, & (x,t)\in\Gamma\times(0,T], \end{cases} \tag{1.1}$$

where $\Omega\in\mathbb{R}^2$ is the bounded domain with sufficiently smooth boundary Γ. $u=u(x,t)$, $p=p(x,t)$ and $f=f(x,t)$ are the velocity, the pressure and the prescribed external force at position $x\in\Omega$ and time $t\in(0,T]$ ($T>0$ is the final time),

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* Corresponding author.
E-mail addresses: zhangtong0616@163.com, zhaojing0216@163.com, mathzhangtong@gmail.com (J. Zhao).

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respectively. $\nu, \lambda \geq 0$, $1/\delta$ and $u_0(x)$ represent the viscosity number, the viscoelastic coefficient number, the relaxation time and the initial velocity.

Problem (1.1) is the generalization of the initial boundary value problem of the Navier–Stokes equations. The existence, uniqueness and continuous dependence of the solutions were studied in [2]. In the aspect of the numerical methods, we just refer to [3,7,14,15,24,26] and the reference therein. In above works, one solved the numerical solutions from a large algebraic system, and the used mixed finite element spaces must satisfy the discrete inf–sup condition. In this paper we extend the second order projection method developed by Shen [19,20] to solve the linearized Oldroyd model (1.1), and establish the corresponding convergence results.

By the projection schemes, problem (1.1) is decoupled into two small linear subproblems, and each subproblem is solved easily than the original one. For instance, the following higher order projection schemes are analyzed in [25]:

$$\begin{cases} \dfrac{\widetilde{u}^{n+1} - u^n}{\Delta t} - \nu\Delta\widetilde{u}^{n+\frac{1}{2}} + (u^n \cdot \nabla)\widetilde{u}^{n+\frac{1}{2}} + \nabla\phi^n - \Delta t\lambda \sum_{i=1}^{n} e^{-\delta(t_n - t_i)}\Delta u^i = f(t_{n+\frac{1}{2}}), \\ \widetilde{u}^{n+\frac{1}{2}}|_\Gamma = 0, \end{cases} \tag{1.2}$$

and

$$\begin{cases} \dfrac{u^{n+1} - \widetilde{u}^{n+1}}{\Delta t} + \alpha_1\nabla(\phi^{n+1} - \phi^n) = 0, \\ \nabla \cdot u^{n+1} = 0, \\ u^{n+1} \cdot \vec{n}|_\Gamma = 0, \end{cases} \tag{1.3}$$

where $\Delta t > 0$ is the time step, $\vec{n}$ is the normal vector to Γ, $t_{n+\frac{1}{2}} = (n + \frac{1}{2})\Delta t$, $\widetilde{u}^{n+\frac{1}{2}} = \frac{1}{2}(\widetilde{u}^{n+1} + u^n)$ and $\alpha_1 > \frac{1}{2}$.

Set

$$H = \{u \in L^2(\Omega)^2, \nabla \cdot u = 0, u \cdot n|_\Gamma = 0\},$$

and define P_H is the projector from $L^2(\Omega)^2$ onto H, i.e.,

$$(u - P_H u, v) = 0, \ \ \forall u \ \in Y, \ \ v \in H, \tag{1.4}$$

then, we can check that (1.3) is equivalent to $u^{n+1} = P_H\widetilde{u}^{n+1}$, which explains why we call (1.2) and (1.3) as the projection schemes. In [25], we established the convergence of weakly second order for velocity and of weakly first order for pressure. In this paper, instead of the projection schemes (1.2) and (1.3), we consider the following numerical schemes for problem (1.1):

Step 1: Start with $(u^0, p^0) = (u(t_0), p(t_0)) \in H_0^1(\Omega)^2 \times (H_0^1(\Omega)/\mathbb{R})$ find u^1 satisfying

$$\begin{cases} \dfrac{u^1 - u^0}{\Delta t} - \nu\Delta u^{\frac{1}{2}} + (u^0 \cdot \nabla)u^{\frac{1}{2}} + \nabla p^0 = f(t_{n+\frac{1}{2}}), \\ u^0|_\Gamma = 0. \end{cases} \tag{1.5}$$

Step 2: Using u^1 obtained from (1.5) to find $p^1 \in H_0^1(\Omega)/\mathbb{R}$ as a solution of

$$\begin{cases} \nabla \cdot u^1 - \alpha\Delta t(\Delta p^1 - \Delta p^0) = 0, \\ \dfrac{\partial p^1}{\partial \vec{n}}|_\Gamma = \dfrac{\partial p^0}{\partial \vec{n}}|_\Gamma. \end{cases} \tag{1.6}$$

Step 3: Start with $(u^1, p^1) \in H_0^1(\Omega)^2 \times (H_0^1(\Omega)/\mathbb{R})$ (or the obtained solutions (u^n, p^n)), find $u^{n+1} (n \geq 1)$ satisfying

$$\begin{cases} \dfrac{u^{n+1} - u^n}{\Delta t} - \nu\Delta u^{n+\frac{1}{2}} + (u^n \cdot \nabla)u^{n+\frac{1}{2}} + \nabla p^n - \Delta t\lambda \sum_{i=1}^{n} e^{-\delta(t_n - t_i)}\Delta u^i = f(t_{n+\frac{1}{2}}), \\ u^{n+1}|_\Gamma = 0. \end{cases} \tag{1.7}$$

Step 4: Using u^{n+1} in step 3 to find $p^{n+1} \in H_0^1(\Omega)/\mathbb{R}$ as a solution of

$$\begin{cases} \nabla \cdot u^{n+1} - \alpha\Delta t(\Delta p^{n+1} - \Delta p^n) = 0, \\ \dfrac{\partial p^{n+1}}{\partial \vec{n}}|_\Gamma = \dfrac{\partial p^n}{\partial \vec{n}}|_\Gamma, \end{cases} \tag{1.8}$$

where $u^{n+\frac{1}{2}} = \frac{1}{2}(u^{n+1} + u^n)$ and the constant $\alpha > \frac{1}{4}$.

In principle, our numerical schemes at the time level t_{n+1} could be designed as follows:

(I) Compute u^1 from (1.5) and update p^1 from (1.6).
(II) Based on (u^1, p^1), computer u^{n+1} from (1.7) and update p^{n+1} from (1.8).

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