Applied Mathematics and Computation 316 (2018) 390–399

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Strategy optimization for static games based on STP method[☆]

Liqing Wang[a], Yang Liu[a,b,*], Zhengguang Wu[c], Fuad E. Alsaadi[d]

[a] College of Mathematics, Physics and Information Engineering, Zhejiang Normal University, Jinhua, China
[b] Department of Automation, Zhejiang University of Technology, Hangzhou 310023, China
[c] National Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, China
[d] Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Keywords:
Strategy optimization
Nash equilibrium
Dominant strategy
Dominated strategy
Semi-tensor product

ABSTRACT

In this paper, we consider the choice behavior of players in a static game and discuss solutions to static games of complete information problem via semi-tensor product (STP) method. Some properties of the dominant behaviors are obtained, based on which, choosing the dominant strategies deleting the dominated strategies are used to optimize the strategy sets so as to obtain solutions to the considered game. Examples are given to show the effectiveness of the results.

1. Introduction

Game theory is a mathematical theory concerned with the optimum choice of strategy in situations involving a conflict of interest pioneered by John von Neumann [1]. Evolutionary game [2], including coevolutionary game, [3] will no longer model into a rational game, but achieve the game equilibrium by trial and error method. It has wide real-world applications in many fields, including economics, biology, computer science, aerospace, and so on. The Nash equilibrium is a fundamental concept in game theory, which means that each player has made his or hers stable choice. In [4] Nash proved that if mixed strategies were allowed, then every game with a finite number of players and strategies had at least one Nash equilibrium. It has been shown that every congestion game and every finite potential game possess a pure Nash equilibrium (PNE) in [5,6].

According to the difference of the decision order made among the players, a game can be divided into static case and dynamic case [7]. A static game means that all players take actions at the same time or if they do not take actions together, they know nothing about what others have chosen. A dynamic game means that players making decisions in order and players can get part or complete information of the game's history. Besides, a game can also be distinguished as cooperative or non-cooperative [8], complete information or incomplete information [9]. One of the most classic game models in game theory is Prisoner's Dilemma [10], which is a non-cooperative, complete information and static game.

A static game of complete information is a fundamental kind of games, and the method of finding a solution to such game is called List Method [9]. Such method is easy to understand and convenient, but powerless when facing big data.

☆ This work was partially supported by the National Natural Science Foundation of China under Grant 11671361, the China Postdoctoral Science Foundation under grant 2016T90406 and 2015M580378, and the Open Foundation of Control Science and Engineering Disciplines at Zhejiang University of Technology under grant 20170901.

* Corresponding author at: College of Mathematics, Physics and Information Engineering, Zhejiang Normal University, Jinhua, China.
E-mail address: liuyang@zjnu.edu.cn (Y. Liu).

http://dx.doi.org/10.1016/j.amc.2017.08.023
0096-3003/

Cheng et al. [11] developed a novel method, called semi-tensor product (STP). More details can also refer to [12]. Based on STP, there have been many results on Boolean networks and Boolean control networks, including fixed points and cycles [13], reachability [14,15], controllability [16–18], observability [19,20] and stabilization [21,22,24–26]. Moreover, system decomposition [27,28], output regulation [15,29], synchronization problems [30,31] and disturbance problems [32] have also been investigated. Pinning control [33–35], feedback control [22,23], and output tracking control [36] using the STP method are designed for BCNs. Liu et al. [37,38] also investigate singular BNs using STP method. As another application of STP, Cheng et al. [39] has applied it to networked evolutionary games, including analysis on the dynamics and Nash equilibrium [40], finding the potential function of the potential games [41], stability analysis for strategy profile of the evolutionary network games [42].

In this paper, we investigate the algebraic formulation and strategy optimization for strategy sets of a game by STP. The main contributions are as follows. (1) The STP method is applied to the discussion of static games with complete information. Besides, a new mathematic representation of the game is established. (2) The process of the game is converted to an algebraic form, based on which, the dominant strategy and the dominated strategy have their mathematical expressions. (3) The strategy sets are optimized and a solution to the game is obtained. Moreover, an algebraic expression of the Nash equilibrium is presented.

The remainder paper is organized as follows. Section 2 gives some necessary preliminaries. Section 3 analyzes some properties of dominant strategies and dominated strategies in STP form, based on which an algebraic expression of Nash equilibrium and a method to optimize the strategy sets are obtained. Section 4 is a brief conclusion.

2. Preliminaries

We first list some notations:

(1) $\mathcal{M}_{m\times n}$ is the set of all $m \times n$ real matrices.
(2) $Col_i(M)$ $(Row_i(M))$ is the ith column (row) of matrix M; $Col(M)$ $(Row(M))$ is the set of columns (rows) of M.
(3) $\delta_n^i = Col_i(I_n)$ is the ith column of the identity matrix.
(4) $\Delta_n = Col(I_n)$.
(5) Assume $L = [\delta_m^{i_1}, \delta_m^{i_2}, \ldots, \delta_m^{i_n}]$, then its shorthand form is $L = \delta_m[i_1, i_2, \ldots, i_n]$.
(6) $V_r(A)$ is the row stacking form of matrix A, and it can be expressed by $V_r(A) = (a_{11}, a_{12}, \ldots, a_{1n}, a_{21}, \ldots, a_{2n}, \ldots, a_{n1}, \ldots, a_{nn})$.

Definition 1. [11] Let $A \in \mathcal{M}_{m\times n}$ and $B \in \mathcal{M}_{p\times q}$. Denote by $t = lcm(n, p)$ the least common multiple of n and p. Then we define the semi-tensor product (*STP*) of A and B as

$$A \ltimes B = (A \otimes I_{t/n})(B \otimes I_{t/p}) \in \mathcal{M}_{mt/n\times qt/p},$$

where $\otimes$ is the tensor (or Kronecker) product.

When $n = p, A \ltimes B = AB$. Therefore the STP is a generalization of the conventional matrix product. So symbol "$\ltimes$" can be omitted, and the matrix product is assumed to be STP in the sequel.

Proposition 1. *[11] Let $x \in R^t$ be a column vector. Then for a matrix M*

$$xM = (I_t \otimes M)x.$$

Definition 2. [11] Define a matrix:

$$W_{[n,m]} := [\delta_n^1\delta_m^1, \delta_n^2\delta_m^1, \ldots, \delta_n^n\delta_m^1; \delta_n^1\delta_m^2, \ldots, \delta_n^n\delta_m^2, \ldots, \delta_n^n\delta_m^n],$$

$W_{[n,m]} \in \mathcal{M}_{mn\times mn}$.

Proposition 2. *[11] Let $X \in R^m$and $Y \in R^n$ be two column vectors. Then*

$$W_{[m,n]}XY = YX.$$

Definition 3. [11] Assume $X \in \Delta_p$, $Y \in \Delta_q$, we define two dummy matrices: (1) $D_f^{[p,q]}$, named by front-maintaining operator (FMO) and (2) $D_r^{[p,q]}$, named by rear-maintaining operator (RMO), respectively, as follows:

$$D_f^{[p,q]} = I_p \otimes 1_q^T, \quad D_r^{[p,q]} = 1_p^T \otimes I_q.$$

Then we have

$$D_f^{[p,q]}XY = X, \quad D_r^{[p,q]}XY = Y.$$