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Applications of Quintic Hermite collocation with time discretization to singularly perturbed problems

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ABSTRACT

Singular perturbation problems have been discussed using collocation technique with quintic Hermite interpolating polynomials as base functions. These polynomials have the property to interpolate the function as well as its tangent at node points. To discretize the problem in temporal direction forward difference operator has been applied. The given technique is a combination of collocation and difference scheme. Parameter uniform convergence has been studied using the method given by Farrell and Hegarty (1991). Rate of convergence of quintic Hermite difference scheme has been found to depend upon node points. Applicability and computational effect of the scheme has been examined through numerical examples. Results have been presented graphically through surface plots as well as in tabular form.

1. Introduction

Mathematical modeling of various processes in physics, chemistry, mathematics etc. can be described via differential equations. Singular perturbation problems can be defined as those problems in which the perturbation parameter is multiplied to highest order derivative. These problems also have an important place in the category of boundary value problems. Singular perturbation basically, shows the nature of the differential equation which changes rapidly as perturbation parameter approaches to zero. Burgers equation [1,2], Kuramoto–Sivashinsky equation [3,4], Neuronal's model [5] are some examples of non linear and linear singular perturbation problems. The singularly perturbed nature of these problems describes the physical phenomena of boundary layers. The concept of singular perturbation becomes more relevant when diffusion term is smaller than convection term. It is equivalent to the fact that singular perturbation parameter should strictly lie between 0 and 1. The study of singular perturbation problems depends upon its solution and error analysis to analyze the rate of convergence. Consider the following form of linear singular perturbation problem:

$$\frac{\partial u}{\partial t} = \Im_\varepsilon(u) + f(x,t), \quad 0 < x < 1, t > 0 \tag{1}$$

$$\Im_\varepsilon \equiv \varepsilon\frac{\partial^2}{\partial x^2} - p(x)\frac{\partial}{\partial x} - q(x),$$

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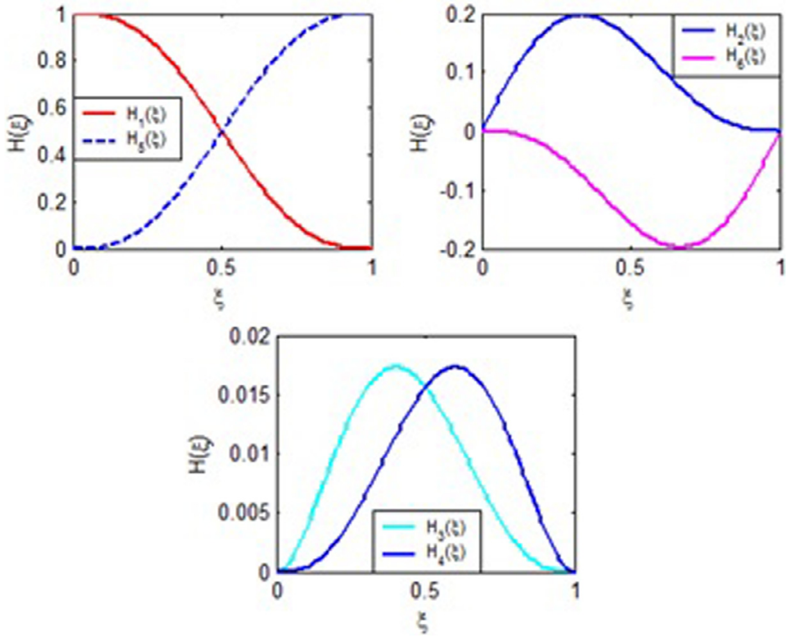

Fig. 1. Graphical representation of Quintic Hermite interpolating polynomials.

$p(x) \geq 0$ and $q(x) \geq 0$ are sufficiently smooth functions on [0,1]. Robin′s mixed boundary conditions have been taken as:

$$a_1 u + a_2 \frac{\partial u}{\partial x} = 0, \quad at\ x = 0 \tag{2}$$

$$a_3 u + a_4 \frac{\partial u}{\partial x} = 0, \quad at\ x = 1 \tag{3}$$

a_1, a_2, a_3 and a_4 are constants, where a_1, a_2 and a_3, a_4 are not both zero. Initially, $u(x, 0) = u_0(x)$.

From the last few years a great deal of efforts has been done to find the solutions (analytic as well as numeric) of singular perturbation problem using Laplace transform [6–9], homotopy perturbation method [10–15], finite difference scheme [16–20], Galerkin method [21,22], collocation [23–27], and spline functions [5,28–32] etc. In the present study, technique of quintic Hermite collocation has been followed to discretize the given singular perturbation problem.

2. Quintic Hermite collocation method (QHCM)

Quintic Hermite collocation method is one of the Hermite collocation method [33,34] where Hermite interpolating polynomials are used as basis in collocation technique [24–26].

Collocation is one of the weighted residual method in which an unknown function is assumed to approximate the differential equation and the boundary adjoining it. Trial function is approximated using Lagrangian interpolating polynomial which discretizes the unknown function and can be represented by a series of orthogonal polynomials. The residual is defined over its region and is set equal to zero at collocation points.

In Hermite collocation trial function is approximated by Hermite interpolating polynomials of order $2k+1(k > 0)$. It is the generalization of Lagrange interpolation with polynomials that not only interpolate function at each node but also its consecutive derivatives. Due to this property QHCM gives better results for mixed boundary conditions.

In general for real numbers $x_1 < x_2 < x_3 < \ldots < x_k$ and all integers $m_1, m_2, m_3, \ldots, m_k$ greater than zero there exist a unique polynomial of degree $m_1 + m_2 + m_3 + \cdots + m_k - 1$. In present study quintic Hermite polynomials of order 5 have been taken as base function to approximate the trial function. The behavior of quintic Hermite interpolating polynomials have been shown graphically in Fig. 1. The structure of elements in QHCM using quintic Hermite interpolating polynomials is explained in Fig. 2. The boundary conditions are defined at x=0 and x=1. Zeros of Legendre polynomial has been taken as collocation points.

3. Error analysis

Maximum principle [35]: Let $u(x)$ be the solution of advection-diffusion equation such that $u(0) \geq 0$ and $u(1) \geq 0$. Then $\mathfrak{L}u(x) \geq 0$, $\mathfrak{L}$ being differential operator, for all x in domain $\Omega(0,\ 1)$ implies that $u(x) \geq 0$ for all x in $\bar{\Omega}$.