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Promoting cooperation by punishing minority

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ABSTRACT

Punishment is an effective way to sustain cooperation among selfish individuals. In most of previous studies, objects of punishment are set to be defectors. In this paper, we propose a mechanism of punishment, in which individuals with the majority strategy will punish those with the minority strategy in a public goods game group. Both theoretical analysis and simulation show that the cooperation level can be greatly enhanced by punishing minority. For no punishment or small values of punishment fine, the fraction of cooperators continuously increases with the multiplication factor. However, for large values of punishment fine, there exists a critical value of multiplication factor, at which the fraction of cooperators suddenly jumps from 0 to 1. The density of different types of groups is also studied.

1. Introduction

To understand the emergence of cooperative behavior among selfish individuals, researchers have considered various mechanisms [1–3], such as network reciprocity [4–6], voluntary participation [7,8], teaching activity [9–11], social diversity [12], migration [13,14], chaotic payoff variations [15], extortion [16–18], reputation [19], memory [20,21], diverse activity patterns [22], the coevolution setup [23], onymity [24] and so on.

So far, punishment has been proved to be an effective way to enforce cooperative behavior and various mechanisms of punishment have been proposed [25–32]. Szolnoki et al. found that the impact of pool punishment on the evolution of cooperation in structured populations is significantly different from that reported previously for peer punishment [33]. Perc and Szolnoki proposed an adaptive punishment that can promote public cooperation through the invigoration of spatial reciprocity, the prevention of the emergence of cyclic dominance, or the provision of competitive advantages to those that sanction antisocial behavior [34]. Chen et al. showed that sharing the responsibility to sanction defectors rather than relying on certain individuals to do so permanently can solve the problem of costly punishment [35]. Cui and Wu demonstrated that the presence of selfish punishment with avoiding mechanism can help individuals out of both first-order and second-order social dilemma [36].

In previous studies, objects of punishment are individuals who hold a specific strategy (usually is deemed to be defection). However, the punished strategy may not be fixed but depends on the surrounding environment, e.g., on neighbors' strategies. Psychological experiments have demonstrated that, an individual tends to follow the majority in behavior or opinion. There exist some psychological or financial punishments of being minority [37,38]. Based on the above consideration,

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we propose a mechanism of punishment in which individuals with the majority strategy will punish those with the minority strategy in a group. Utilizing the public goods game (PGG) as a prototypical model of group interaction, we find that cooperation can be greatly promoted by punishing minority.

2. Model

Our model is described as follows.

Individuals are located on a 1000×1000 square lattice with periodic boundary conditions. Every individual occupies a lattice point and has four neighboring points. A PGG group is composed of a sponsor and its four neighbors. Thus, the size of each PGG group is 5. Each individual i participates in five different PGG groups organized by i and its four neighbors, respectively.

At each time step, every cooperator contributes a unit cost to each involved PGG group. Defectors invest nothing. The total cost of a group is multiplied by a factor r, and is then redistributed uniformly to all the five players in this group. In every PGG group, individuals with the majority strategy will punish those with the minority strategy. Each minority in the group is punished with a fine α. Following the previous study [35], we assume that punishers equally share the associated costs. If cooperators are majority and defectors are minority in the group, then each cooperator pays a cost $\alpha(5-n_c)/n_c$, where n_c is the number of cooperators in the group. Oppositely, if defectors are majority and cooperators are minority in the group, then each defector pays a cost $\alpha n_c/(5-n_c)$.

We denote i's strategy as $s_i = 1$ for cooperation and $s_i = 0$ for defection. The payoff that player i gains from the group organized by player j is calculated by the following equations:

$$if\ n_c = 0 \text{ or } 5,\ \ \Pi_i^j = -s_i + \frac{rn_c}{5}; \tag{1}$$

$$if\ 2 < n_c < 5 \text{ and } s_i = 1,\ \ \Pi_i^j = -1 + \frac{rn_c}{5} - \frac{\alpha(5-n_c)}{n_c}; \tag{2}$$

$$if\ 2 < n_c < 5 \text{ and } s_i = 0,\ \ \Pi_i^j = \frac{rn_c}{5} - \alpha; \tag{3}$$

$$if\ 0 < n_c < 3 \text{ and } s_i = 1,\ \ \Pi_i^j = -1 + \frac{rn_c}{5} - \alpha; \tag{4}$$

$$if\ 0 < n_c < 3 \text{ and } s_i = 0,\ \ \Pi_i^j = \frac{rn_c}{5} - \frac{\alpha n_c}{5-n_c}. \tag{5}$$

Eq. (1) means that there is no punishment when the group is occupied by full cooperators or full defectors. For Eqs. (2) and (3), cooperators punish defectors in the group. While for Eqs. (4) and (5), defectors punish cooperators in the group. The total payoff of the player i is calculated by

$$P_i = \sum_{j\in\Omega_i} \Pi_i^j, \tag{6}$$

where Ω_i denotes the community of neighbors of i and itself.

Initially, cooperators and defectors are randomly distributed with the equal probability 0.5. Individuals asynchronously update their strategies in a random sequential order [39–41]. Firstly, an individual i is randomly selected who obtains the payoff P_i according to the above equations. Next, individual i chooses one of its nearest neighbors at random, and the chosen neighbor j also acquires its payoff P_j. Finally, individual i adopts the neighbor j's strategy with the probability [42]:

$$W(s_i \leftarrow s_j) = \frac{1}{1+\exp[(P_i - P_j)/K]}, \tag{7}$$

where K characterizes the noise introduced to permit irrational choices. Following the previous studies [43,44], we set the noise level to be $K = 0.5$.

The key quantity for characterizing the cooperative behavior of the system is the fraction of cooperators ρ_c in the steady state. In our simulation, ρ_c is obtained by averaging over the last 10^4 Monte Carlo steps (MCS) of the entire 10^6 MCS. Each MCS consists of on average one strategy-updating event for all individuals. Each data is obtained by averaging over 20 different realizations.

3. Results

Fig. 1 shows the fraction of cooperators ρ_c as a function of the multiplication factor r for different values of the punishment fine α. From Fig. 1, one can see that, for no punishment ($\alpha = 0$) or a light punishment (e.g., $\alpha = 0.3$), ρ_c continuously increases from 0 to 1 as r increases. However, for a severe punishment (e.g., $\alpha = 0.8$), there exists an abrupt transition point ($r \approx 3.38$ for $\alpha = 0.8$), at which ρ_c suddenly jumps from 0 to 1. For a fixed value of r, the payoff difference between the two