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A non-Newtonian liquid sphere embedded in a polar fluid saturated porous medium: Stokes flow

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ABSTRACT

The selection of interface boundary conditions between porous-medium and clear-fluid regions is vital for the extensive range of applications in engineering. As such, present paper reports an analytically investigating Stokes flow over Reiner–Rivlin liquid sphere embedded in a porous medium filled with micropolar fluid using Brinkman's model and assuming uniform flow away from the obstacle. The stream function solution of Brinkman equation is obtained for the flow in porous region, while for the inner flow field the solution is obtained by expanding the stream function in a power series of *S*. The flow fields are determined explicitly by matching the boundary conditions at the interface of porous region and the liquid sphere. Relevant quantities such as velocity and pressure on the surface of the liquid sphere are determined and exhibited graphically. The mathematical expression of separation parameter *SEP* is also calculated which shows that no flow separation occurs for the considered flow configuration and also validated by its pictorial presentation. The drag coefficient experienced by a liquid sphere embedded in a porous medium is evaluated. The useful features of the Stokes flow for numerous values of parameters are analyzed and discussed. The dependence of the drag force and stream line pattern on permeability parameter(η^2), viscosity ratio(λ), micropolar parameter(m), coupling number(N), and dimensionless parameter S is presented graphically and discussed. The analysis also aims at the explanation of velocity overshoot behavior. Some previous noted results are then also obtained from the ongoing analysis.

1. Introduction

In view of analytic and numerical techniques and methods, many investigators have taken into consideration the study of uniform motion of liquids past and through the porous media with regard to different geometries and their orientation which has numerous engineering and industrial applications. Many investigations of liquid motions over and within porous bodies are restricted prominently to very low/small Reynolds numbers. Several authors have used many different experimental and theoretical models for the analysis of flows in a porous medium considering Darcy's model. The liquid motion of an incompressible Newtonian fluid over a porous spherical body was investigated by Joseph and Tao [1] using the Darcy's law with the condition of no-slip at sphere's surface and found the same drag as on the rigid body with minimized radius. Using Saffman's boundary condition at the surface of the porous cylinder, Palaniappan et al. [2] examined the fluid motion over a porous right circular cylinder with the application of Darcy's model. Vanshtein et al. [3] examine the fluid phenomenon over and through the porous spheroid by using Darcy's law with the condition of continuity of tangential ve-

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Table 1
Following is the table showing some of the related important investigations regarding present work.

Models	Author(s)/Year
• A body embedded in a saturated porous medium	Srinivasacharya and Murthy (2002)[14]
• A spherical porous sphere embedded in another porous medium	Grosan and Postelnicu (2010)[15]
• A fluid sphere embedded in porous medium	Deo et al. (2010)[16]
• A porous sphere embedded in another porous medium	Deo and Gupta (2010)[17]
• A porous spheroid embedded in another porous medium	Yadav and Deo (2012)[18]
• An inclined wavy surface embedded in a nano fluid saturated porous medium	Srinivasacharya and Kumar (2015)[19]
• A Reiner–Rivlin liquid sphere immersed in a saturated porous medium	Jaiswal and Gupta (2015)[20]

locity at the surface of the spheroid. They have also obtained some limiting cases of porous circular disk and elongated rods. Several researchers have solved the problems of flow past a body embedded in a porous medium adopting Darcy's model. However, for high porosity and comparatively large shear rates for the flows in porous medium, the Darcy's law seems to be insufficient. Therefore, to overcome such intricacy, Brinkman [4] and Debye and Beuche [5] suggested a modification to Darcy's law as

$$\tilde{\mu}\nabla^2\mathbf{q} - \frac{\mu}{k}\mathbf{q} = \nabla p, \tag{1}$$

generally known as the Brinkman's equation. Here μ and $\tilde{\mu}$, respectively, are viscosity of clear fluid and effective viscosity of porous region, k stands for the permeability of the porous medium and $\mathbf{q}$ is the velocity of the fluid. Using this improved model, Qin and Kaloni [6] found a Cartesian-tensor solution of the fluid flow over a permeable sphere and also evaluated the hydrodynamic force experienced by a porous sphere. The problem of creeping flow past porous particles using this modified form (1) was studied by Higdon and Kojima [7] and derived some useful results of asymptotic behavior for low and high porosity using Greens function. Nandkumar and Masliyah [8] solved the problem of flow past an isolated permeable sphere using the Darcy's extension law within the porous sphere and solved the Navier–Stokes equation using a finite difference scheme. The problem of an incompressible viscous flow over a circular cylinder embedded in a constant porosity medium based on the Brinkman model was investigated by Pop and Cheng [9] and obtained a closed form of exact solution of stream function of Brinkman's equation. The problem of flow of a viscous fluid over a solid sphere immersed in a medium of uniform porosity was solved by Barman [10] and obtained a closed form to the governing equations using stream function. Problem of flow past a sphere embedded in a porous medium based on the Brinkman model is also solved by Pop and Ingham [11] and presented exact solution for the forced flow which shows that no flow separation takes place for the present flow configuration. Rudraiah et al. [12] have solved the problem of flow past an impervious sphere embedded in a constant and high porosity porous medium using non-Darcy model and obtained an exact solution for the governing equation specifying a constant shear away from the sphere. Ganapathy [13] has presented an analytical study of the problem of creeping flow past a solid sphere in porous medium assuming the validity of the Brinkman model and obtained a closed form solution for the flow field. The above Table 1 shows some important previous investigations related to present work.

In all of the above studies the fluid is assumed to be Newtonian. The hydrodynamics of classical fluids is based on the assumption that the fluid particles do not have any internal structure. However, there are some fluids such as polymeric suspensions, melts, drilling mud, clastomers, animal blood, certain oils and greases and more emulsions are classified as non-Newtonian fluids in which fluid particles may exhibit some microscopical effects such as rotation, shrinking etc. Therefore, the internal structure should be taken into account for fluids whose particles have complex shapes. The mathematical model of micropolar fluid introduced by Eringen [21] enables us to study many physical phenomena arising from the local structure and micro-motions of the fluid particles. Hoffmann et al. [22] introduced a new variable τ which accounts the spin motion of microrotations of fluid particles.

Srinivasacharya and Rajyalakshmi [23] researched the creeping flow of incompressible micropolar fluid past a porous sphere with zero-spin condition for microrotations. The numerical solution of the flow past an impermeable sphere embedded in a fluid-saturated porous medium using the Forchheimer–Brinkman–Darcy extended model was recently investigated by Juncu [24]. In his investigation, he obtained that with the decrease in Darcy number (Da) vanishes the influence of the macroscopic inertial terms and the solutions of the generalized mathematical model equations approaches to the slow viscous flow solutions. The problem of mixed convection over a irregular surface embedded in a nanofluid saturated porous medium was investigated by Srinivasacharya and Kumar [19] and found that the effect of the amplitude of the wavy surface is to enhance the velocity, temperature, nanoparticle volume fraction of aiding flow and the reverse trend is observed for opposing flow.

The generalization of the linear relationship between stress and rate of strain tensor is made non-linear from the general idea of fluidity by Stokes [25], and then approximating it to suit a particular fluid was done by Reiner [26] using Cayley–Hamilton theorem for matrices of rank 3. Rivlin [27] also obtained the similar equation by assuming that stress is a function of the velocity gradients. So, for an incompressible and isotropic fluid to depict the conduct of wet sand, Reiner and Rivlin derived a particular constitutive equation

$$\tilde{\tau}_{ij} = -\tilde{p}\delta_{ij} + \mu_2\tilde{d}_{ij} + \mu_c\tilde{d}_{ik}\tilde{d}_{kj} \tag{2}$$

and the fluid governed by this equation is known as the Reiner–Rivlin fluid.