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A fractional epidemiological model for computer viruses pertaining to a new fractional derivative

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ABSTRACT

In the computer security and for any defensive strategy, computer viruses are very significant aspect. To understand their expansion and extension is very important component. In order to deal with this issue, we consider a fractional epidemiological model. In this article, we analyze moderate epidemiological model to describe computer viruses with an arbitrary order derivative having non-singular kernel. We obtain the solution of the problem by using an iterative method. By using the fixed-point theorem the existence of the solution is discussed. The uniqueness of the solution is verified. We perform some numerical simulations and show graphically to observe the impact of the arbitrary order derivative.

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1. Introduction

The computer virus is a big threat as hardware and software technology advanced and computer network becomes an immense device for day-to-day life. It is a type of computer program which reproduce itself and extend from one computer to others. The file system mostly attacked by the viruses and worms use system vulnerability to search and attack computers. Hence, for upgrading the security and reliability in the computer system and networks, the test on better investigation of the computer virus propagation dynamics is very crucial subject. There are basically two ways to investigate this problem same to the biological virus as microscopic and macroscopic models. Kephart et al. [1,2] initiated first towards modeling the expand characteristic of computer virus by following the macroscopic technique. Billings et al. [3] investigated a uniform prediction of computer virus spread inconnected networks. Han and Tan [4] analyzed dynamical response of computer virus on Internet. In a study Piqueira and Araujo [5] discussed a modified epidemiological model (MEM) for computer viruses. Ren et al. [6] studied a new computer virus model and its dynamic. Wierman and Marchette [7] explained the modeling of computer virus. Handam and Ferihat [8] formulated a fractional MEM for computer viruses.

Since 1988 epidemic models describing computer virus spread have been demonstrated. To describe the connection between epidemiology and computer viruses Murray [9] seems to be first. However Murray did not explain any particular model, he demonstrated analogies to some public health epidemiological defense strategies. In another study Gleissner [10] investigated a model of computer virus spreading on multi-user system. Though, he did not explain for the detection and elimination of viruses or altering other users of the existence of virus. Latterly, Kephart et al. [1,2,11,12], a group

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at International Business Machines Watson Research center has demonstrated susceptible-infected- susceptible (SIS) models describing computer virus spreading.

Kephart et al. [12] introduced a direct random graph model and discussed its characteristic. Lu et al. [13] studied the effect of constant and pulse vaccination of SIR epidemic model having horizontal and vertical transmission. Kephart et al. [1,2] claimed that a mixture of theory and observation suggest that computer viruses were much less prevalent than many have explained. They further cited that the number of infected machines is perhaps 3 or 4 per 1000 PC's estimated and said that because of the number of viruses is increasing with time, not because of any single viral strain, computer viruses are slowly becoming more widespread. Piqueira and Araujo [5], Wierman and Marchette [7] introduced the standard SIR computer virus propagation model that is expressed as follows:

$$\begin{aligned}\frac{dS}{dt} &= b - \lambda S(t)I(t) - dS(t),\\ \frac{dI}{dt} &= \lambda S(t)I(t) - \varepsilon I(t) - dI(t),\\ \frac{dR}{dt} &= \varepsilon I(t) - dR(t).\end{aligned} \tag{1}$$

In the above Eq. (1) $S(t)$ is indicating the numbers of susceptible computers at time t, $I(t)$ is denoting the numbers of infected computers at time t, and $R(t)$ is denoting the numbers of recovered computers at time t, b is standing for the rate at which external computers are connected to the network, ε is representing the recovery rate of infected computer because of the anti-virus ability of the network, d is denoting the rate at which one computer is removed from the network and λ is standing for the rate at which, when having a connection to one infected computer, one susceptible computer can become infected.

Latterly, Ren et al. [6] proposed a newly recovery function as follows

$$T(I) = \begin{Bmatrix} \varepsilon I, 0 \le I \le I_0 \\ m, I > I_0 \end{Bmatrix}, \tag{2}$$

where ε indicates the recovery rate if the anti-virus ability is not fully used, characterize the saturation phenomenon of the limited anti- virus ability of a network is presented by $m = \varepsilon I_0$. Further they [6] attentively analyzed the dynamics of the computer virus propagation model given as follows

$$\begin{aligned}\frac{dS}{dt} &= rS\left(1 - \frac{S}{k}\right) - \lambda SI - dS,\\ \frac{dI}{dt} &= \lambda SI - T(I) - dI,\\ \frac{dR}{dt} &= T(I) - dR.\end{aligned} \tag{3}$$

Fractional calculus has been used to model the real world problems. It is playing a very important role in the field of science, engineering, finance etc. In the branch of fractional calculus, fractional derivatives and fractional integrals are important aspects. Nowadays, many researchers and scientists investigated in this special branch [14-17]. Atangana and Alkahtani [18] studied the Keller-Segel model involving a fractional derivative having non-singular kernel. Atangana and Alkahtani [19] analyzed the non-homogenous heat model. Singh et al. [20] investigated a fractional biological population model. Kumar et al. [21] explained the fractional reaction-diffusion equations. Choudhary et al. [22] investigated the fractional order differential equations occurring in fluid dynamics. Kumar et al. [23] considered a non-integer model of differential-difference equation having applications in nanotechnology. Kumar et al. [24] analyzed a logistic equation pertaining to a new non-integer derivative possessing non-singular kernel. Yang et al. [25] discussed a fractal LC-electric circuit modeled by local fractional calculus. In another study Yang [26] proposed a new integral transform operator for solving the heat-diffusion problem. Debbouche and Nieto [27] analyzed an approximate controllability of fractional delay dynamic. Debbouche and Nieto [28] investigated a relaxation in controlled systems. Debbouche and Torres [29] studied the Sobolev type fractional dynamic equations and optimal multi-integral controls. Kumar et al. [30] obtained analytical solution of fractional Navier-Stokes. Sarwar and Rashidi [31] examined the two-term fractional-order diffusion, wave-diffusion, and telegraph models arising in mathematical physics. Abelman et al. [32] considered subordination conditions for a class of non-bazilevivc type defined with the aid of fractional q-calculus operators. Yang et al. [33] studied Klein-Gordon equations on Cantor sets. In a recent study Caputo and Fabrizio [34] proposed a novel fractional derivative having non-singular kernel and in addition Losada and Nieto [35] investigated fundamental properties of the newly fractional derivative.

Motivated by ongoing research in this field, we use the new fractional derivative to the fractional MEM for computer viruses. The principal contribution of this work is using the new fractional derivative in MEM for computer viruses. The exactness and uniqueness of the solution of the fractional model is proved by applying the fixed-point theorem. The foundation of this paper is as follows: In Section 2, we present the basic theory of the Caputo-Fabrizio derivative. In Section 3, the fractional modified epidemiological model and approximate solution pertaining to the new Caputo-Fabrizio fractional derivative is analyzed. In Section 4, the existence and uniqueness of system of solutions is verified by employing the fixed-point theory. In Section 5, results and discussion are given. The Section 6, which is the last section contains the conclusions.

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