Applied Mathematics and Computation 313 (2017) 1–11

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A family of non-uniform subdivision schemes with variable parameters for curve design

Mei-e Fang [a], Byeongseon Jeong [b], Jungho Yoon [c,*]

[a] School of Computer Science and Educational Software, Guangzhou University, China
[b] Institute of Mathematical Sciences, Ewha Womans University, Seoul, South Korea
[c] Department of Mathematics, Ewha Womans University, Seoul, South Korea

ARTICLE INFO

Keywords:
Non-uniform subdivision
Variable parameter sequence
Blending curves
Smoothness
Chamfering algorithm

ABSTRACT

In this paper, we present non-uniform subdivision schemes with variable parameter sequences. A locally different tension parameter is set at each edge of the initial control polygon to control locally the shape of the resulting curve such that the scheme becomes non-uniform. Due to the variable parameters, the scheme can reproduce locally different analytic curves such as conics, Lissajous, trigonometric and catenary curves. Hence blending curves including such analytic components can be successfully generated. We discuss the convergence and smoothness of the proposed non-uniform schemes and present some numerical results to demonstrate their advantages in geometric modeling. Furthermore, as an application, we propose a chamfering algorithm which can be used in designing automobile and mechanical products.

1. Introduction

During last decades, there have been intensive studies on free form geometric shape technologies (e.g., generalized B-splines and T-splines) which solve the limitation of the NURBS representation [4,11,15,18–21,23–25,28,29]. They possess all fundamental properties of polynomial B-splines and NURBS, while avoiding drawbacks of NURBS such as complicated rational form, necessity of weights, imprecise description of commonly used transcendental curves in applications. In particular, generalized B-splines can exactly produce not only free-form curves and conics (as NURBS can do), but also many transcendental curves such as Lissajous, catenary, trigonometric function and hyperbolic function curves, while NURBS cannot do it. That is, geometric models represented by generalized B-splines completely include those which can be generated by NURBS.

This study is concerned with generating smooth curves from initially given control points coupled with tension parameter sequences (see e.g., [11,14,16,17]). The recursive generation of smooth curves is a standard method in geometric modeling due to its numerical stability and simplicity for implementation. There have been intensive studies in the area of subdivision-based modeling. One can find families of subdivision schemes for geometric design and graphics applications [6,7,22,27,30]. But it cannot generate those geometric models generated by NURBS. In this view point, it is worthwhile to generate generalized B-splines by subdivision, which is advantageous for both efficiently generating generalized B-splines and applying

* Corresponding author.
E-mail addresses: fme@hdu.edu.cn (M.-e. Fang), bjeongle@gmail.com (B. Jeong), yoon@ewha.ac.kr (J. Yoon).

http://dx.doi.org/10.1016/j.amc.2017.05.063

subdivision scheme in CAD/CAM. The readers are also referred to the studies [1–3,5,8,12,13,26] or the references therein for non-stationary subdivision schemes reproducing trigonometric and exponential polynomials.

In the paper [11], a generalized subdivision scheme with a tension parameter for curve design has been proposed. It can reproduce generalized B-splines and represent exactly many analytic curves such as Lissajous curves, conics, trigonometric and hyperbolic function curves, catenary curves and helixes which are important shapes in geometric modeling. While in engineering applications, blending curves in which some aforementioned analytic curves are inserted are more commonly used. For example, a torque spring is usually composed of a piece of helix, arcs and others. A tool path is usually composed of several pieces of arcs with different radii and lines. In this regard, the aim of this paper is to further develop a family of non-uniform variable parameter subdivision (hereafter, denoted by NVPS) schemes based on the generalized subdivision scheme in [11]. The subdivision rules of the schemes are defined by tension parameters which may vary for any newly defined value. More specifically, the proposed scheme sets a locally different tension parameter at each edge of the initial control points to control locally the shape of the resulting subdivision curve such that the scheme becomes non-uniform. As a result, blending curves composed of different types of curves, which can be respectively generated by the generalized subdivision scheme with different tension parameters, can be successfully reproduced in whole by the proposed scheme. We show that the NVPS of degree d produces C^η-continuous limit curves with $\eta = \min(d-1, 2)$. Some numerical results are presented to demonstrate the advantages of the proposed schemes in geometric modeling. Lastly, as an immediate application, we present a chamfering algorithm which can be used in designing automobile and mechanical products.

The article is organized as follows. In Section 2, we define subdivision rules and steps of our NVPS schemes. Convergence and smoothness of the proposed schemes are analyzed in Section 3. Some numerical examples are given in Section 4. We especially design a chamfering algorithm for application. Conclusion and future work are described in Section 5.

2. Subdivision schemes

2.1. Subdivision rule

From a given set of control points $\mathbf{P}^0 = \{P_i^0 : i \in \mathbb{Z}\}$ at level zero, a binary stationary uniform subdivision scheme defines recursively new sets of points $\mathbf{P}^{k+1} = \{P_i^{k+1} : i \in \mathbb{Z}\}$ at level $k \geq 0$ formally by

$$\mathbf{P}^{k+1} = S\mathbf{P}^k, \quad k = 0, 1, \ldots.$$

A point of $\mathbf{P}^{k+1}$ is defined by a linear combination of points of $\mathbf{P}^k$, that is,

$$P_i^{k+1} = \sum_{j \in \mathbb{Z}} a_{i-2j} P_j^k,$$

with a fixed set of coefficients $\mathbf{a} = \{a_j : j \in \mathbb{Z}\}$, called the subdivision *mask*. Here P_j^k is the value at refinement level k, attached to the dyadic point $j2^{-k}$. It is usual to assume that only finitely many numbers of a_j are non-zero so that changes in a control point affect only its local neighborhood. One should note that only even (and odd) indices of the mask are involved to evaluate new values P_i^k for even (and odd) i respectively.

A non-uniform subdivision may apply a different mask for any newly defined value. The mask defining the new value at level $k+1$ and location i is denoted by $\mathbf{a}^{k,i} := \{a_j^{k,i} : j \in \mathbb{Z}\}$, hence the scheme is defined by

$$P_i^{k+1} = \sum_{j \in \mathbb{Z}} a_{i-2j}^{k,i} P_j^k. \tag{1}$$

Throughout this paper, we assume that all the masks have the same finite support.

2.2. Non-uniform variable parameter subdivision scheme

In this subsection, we introduce non-uniform variable parameter subdivision (NVPS) schemes for curve design. The subdivision rules of the schemes are defined by tension parameters which may vary for any newly defined value. More specifically, for the purpose of generating blending curves composed of different types of generalized B-spline curves, we extend the generalized subdivision in [11] to the case of variable parameters. The proposed schemes set a tension parameter for each edge of the initial control polygon. Each tension parameter can be used to control locally the shape of the resulting subdivision curve. In what follows, we give a detailed algorithm of the proposed non-uniform subdivision schemes. To this end, we first discuss the NVPS of degree 2.

• *NVPS of degree 2*.

Let $\mathbf{P}^0 = \{P_i^0 : i \in \mathbb{Z}\}$ be a set of initial control points with bounded tension parameters $\mathbf{U}^0 = \{u_i^0 \geq 0 : i \in \mathbb{Z}\}$. The NVPS of degree 2, denoted by S_2, defines a set of the new control points $\mathbf{P}^{k+1} (= S_2\mathbf{P}^k)$ as

$$\begin{aligned} P_{2i}^{k+1} &= \frac{1+2u_i^k}{2(1+u_i^k)} P_i^k + \frac{1}{2(1+u_i^k)} P_{i+1}^k, \\ P_{2i+1}^{k+1} &= \frac{1}{2(1+u_i^k)} P_i^k + \frac{1+2u_i^k}{2(1+u_i^k)} P_{i+1}^k. \end{aligned} \tag{2}$$