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Selective harvesting of two competing fish species in the presence of toxicity with time delay

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ABSTRACT

In this paper, we have considered a two competing fish species with the delay due to gestation, each of which obeys the law of logistic growth and releases a substance toxic to the other. We also consider the selective harvesting of fishes above a certain age or size by incorporating a time delay in the harvesting term. The dynamical behavior of the exploited system is examined in the presence and absence of delay. Boundedness, local and global stabilities are addressed. The possibility of the existence of a dynamic equilibrium and the optimal harvesting policy is studied in the absence of delay. It is shown that the time delay can cause a stable equilibrium to become unstable and even a switching of stabilities. Extensive numerical simulations are performed using analytical results for a hypothetical set of parameter values.

1. Introduction

The harmful effects of toxicants have become a major concern for multispecies fisheries or marine aquaculture from both environmental and economical point of view. As the human needs, growing day by day, the industries are producing a huge amount of toxicants. A major part of these toxic substances and chemicals such as arsenic, lead, cadmium, zinc, copper, iron, mercury, etc. are released in lakes, rivers and oceans, affecting the species living therein [1,2]. River, lake and sea water are often contaminated with toxic chemicals, including toxic metals, oil and synthetic organic chemicals. Toxic contaminants lead to a severe reduction in the diversity of the organisms that live in affected regions. The adverse effects can spread, via the food chain, to fish, birds and mammals that feed on contaminated sea life. Therefore, the Uncontrolled contribution of toxicant to the environment is causing many species to extinct and several others are on the verge of extinction. Again, there are many species in the ocean, which produces a toxin (toxin producing phytoplankton TPP) and toxin released by them may affect the growth of the other species significantly. For example phytoplankton species such as Pseudo-nitzschia sp, Gambierdiscus toxicus, Alexandrium sp, Pfiesteria piscicida are highly toxic in nature and they can significantly reduce the grazing pressure of zooplankton releasing toxic chemicals. TPP can act as a strong mediator of zooplankton feeding rate, and thus playing an important role in the species interaction.

The research of marine plankton ecology is very much important for the survival of our earth. There is a global increase in harmful plankton (HAB) over the past two decades [3–6]. Considerable attention has been directed toward HAB because

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of their negative impact on human health, commercial fisheries, subsistence fisheries, recreational fisheries, tourism and coastal recreation, ecosystem and environment are well known. But till now, the exact bio-physical reason behind harmful plankton and its possible control mechanism are not yet established and require special attention [3,5–10,12,13]. Due to this fact, the experimental as well as mathematical modelling is necessary in this subject.

From both environmental and conservational points of view, the effects of toxic substances on ecological communities have become major problems in the recent decades. Mathematical modelling under the effects of toxic substances on various ecosystems apparently started with the studies of Hallam and Clark [14], Hallam et al. [15,16], Hallam and De Luna [17], De Luna and Hallam [18], Freedman and Shukla [19], Ghosh et al. [20], He and Wang [21], Das et al. [22] and many others.

Mathematical studies on tremendous fluctuations in abundance of many phytoplankton communities is an important subject in aquatic environment. These changes of phytoplankton densities within a marine environment have been affected due to important external factors, including variation of necessary nutrients, physical factors or a combination of these two. The researchers observed another important fact that might affect the growth of another species or of several other species which is known as allelopathy. In 1937, Molisch [23] first introduced the term “allelopathy”. After that Rice [23] extensively applied to phytoplankton communities. The toxicants or chemical compounds released by the marine biological species into the surrounding environment [23,24] that effect of one species on the growth of another. The chemical compounds are known as “allelochemicals”. The allelochemicals produced by the unicellular green alga Chlorella vulgaris, is an autotoxin that limits the size of its own population [10,25] and also inhibits the growth of the planktonic algae Asterionella formosa and Nitzschia frustulum (Bacillareae) [11,26]. A number of properties of a substance produced by Chorella (Chorella) extensively studied by Pratt [25]. We may also refer some other planktonic algae that have species inhibitory to other algae, which are Phormidium, Scenedesmus, Pediastrum, Cosmarium, Aphanizomenon, Micrasterias, Oscillatoria, Pandorina, Nostoc, Cylindrospermum, Mesotaenium, Aukistrodesmus, Anabaena, Microcystis, Ceratium, Asterionella, Haematococcus (a motile green alga), Chlamydomonas, Skeletonema, Olithodiscus, Peridinium, Gymnodinium, Ulva, Chorda, Ceramium, Ascophyllum, Chondrus, Fucus, Enteromorpha, Myriophyllum, Ceratophyllum, Lemna, Cladophora, Pithaphora, Hormotilla, Platydorina, Volvax, Eudorina, Gonium, Botrydium, Thalassiosira, Phaeodactylum, Scytonema etc.

It is also observed that allelochemical are not always harmful to some phytoplankton; sometimes they act as a stimulator to growth. For instance, the green alga, Enteromorpha linza produces allelochemicals which are auto-stimulatory and also stimulatory to the growth of Enteromorpha species [9]. Chlorella vulgaris, a unicellular green alga, produces an autotoxin which regulates its own growth and also inhibits the growth of Asterionella formosa and Nitzschia frustulum [10,11]. Several researchers observed many other examples of toxic inhibition and auxin stimulation in planktonic algae.

Maynard-Smith [27] first presented one of the first mathematical models for the allelopathic interaction between two competing species. After that several researchers studied the model proposed by Maynard-Smith, with the hypothetical functional form for the allelopathic interaction term [13,28–30]. A more realistic allelopathic interaction rate between two phytoplankton species C. polylepis and H. triquetra are extremely studied by Solé et al. [24]. The model proposed by Solé et al. [24] was extended by Bandyopadhyay [13]. In 2002, Chattopadhyay et al. [4] presented a mathematical model based on field observations in the Talsari Digha region of the Bay of Bengal in West Bengal, India and showed that toxin producing plankton may act as a biological control for planktonic blooms.

In nature, time delays occur in almost every biological situation [31] and is assumed to be one of the causes of regular fluctuations on population biomass. In population dynamics, a time delay is introduced when the rate of change of population biomass is not only a function of the present population biomass, but also depends on the past population biomass. Therefore, the time delay can be incorporated in the mathematical population model due to various ways, such as maturation time, capturing time and other reasons. Moreover, existence of time delays is frequently a source of instability in some way. Mukhopadhyay et al. [29] modified the Maynard-Smith [27] to a delay differential equation model. They have considered discreate time lag required for the maturity of the species to produce toxic substances. The authors find that there are no destabilizing effects on the dynamics of the model system due to discrete time lag. Abbas et al. [32] have extended the Mukhopadhyay et al. [29] model and obtained almost time periodic solution of the model system. Bandyopadhyay et al. [12] have also considered a delay differential equation model for toxic and non-toxic phytoplankton interaction within a fluctuating environment by introducing multiplicative white noise terms into the growth rates of two phytoplankton species. Recently, Pal and Mahapatra [33] have presented two new delay mathematical models (toxic inhibitory and toxic stimulatory) for allelopathy in the presence of two phytoplankton species.

On the other hand, there are numerous studies on the effects of harvesting on population growth. In the circumstance of predator–prey model, some studies that treat the populations being harvested as a homogeneous resource include those of Brauer and Soudack [34,35] Dai and Tang [36], Myerscough et al. [37] Chaudhuri [38], Leung [39], Murphy and Smith [40], Palma and Olivares [41], Pal et al. [42], Pal and Mahapatra [43] along with others. But they have not considered stage structure of species. Some of the stage structured models using time delay are considered by Aiello and Freedman [44], Freedman and Gopalsammy [45], Rosen [46], Fisher and Goh [47], Cushing and Saleem [48] and some other authors.

Again selective harvesting [49–53] of fish population is an active field of research. Age or body size of a species is generally considered to be one of the most important significant traits because it correlates with many aspects of its biology, from life history to ecology [54–56,56]. Proper harvesting or age/ size selective harvesting is a very significant methodology that dampen fluctuations by making over compensatory dynamics under compensatory. Combined effect of harvesting and delay on prey–predator system are studied by several researchers [57–59]. Jana et al. [60] has been established a age-selective harvesting model by using the technique of Arino et al. [61]. In the same year Jana et al. [62] presented two