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# Parameter Switching Synchronization

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ABSTRACT

In this paper we show how the Parameter Switching algorithm, utilized initially to approximate attractors of a general class of nonlinear dynamical systems, can be utilized also as a synchronization-induced method. Two illustrative examples are considered: the Lorenz system and the Rabinovich–Fabrikant system.



## 1. Introduction

A number of various synchronization methods have been developed, such as complete or identical synchronization, phase and lag synchronization, generalized synchronization, intermittent lag synchronization, imperfect phase synchronization, almost synchronization and so on (see e.g. [1–3], or [4]).

In this paper a new synchronization-induced method, which is based on the Parameter Switching (PS) algorithm [5], is proposed. It is demonstrated [6–8] that this method can be effectively used for the approximation of attractors of a given nonlinear dynamical system which depends linearly on a real parameter.

Let us consider the following initial value problem (IVP), which models a large class of continuous-time nonlinear autonomous dynamical systems depending on a single real control parameter $p$, such as the Lorenz system, Rösler system, Chen system, Lotka–Volterra system, Rabinovich–Fabrikant system, Hindmarsh–Rose system, Lü system, classes of minimal networks and many others, in the following form

$$\dot{x}(t) = f(x(t)) + pAx(t), \quad x(0) = x_0, \tag{1}$$

where $t \in I = [0, T]$, $x_0 \in \mathbb{R}^n$, $p \in \mathbb{R}$ the control parameter, $A \in \mathbb{R}^{n\times n}$ a constant matrix, and $f : \mathbb{R}^n \to \mathbb{R}^n$ a nonlinear function.

For example, we can consider the IVP with $n = 3$ for the Lorenz system

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1), \\ \dot{x}_2 &= x_1(\rho - x_3) - x_2, \\ \dot{x}_3 &= x_1 x_2 - \beta x_3, \end{aligned} \tag{2}$$

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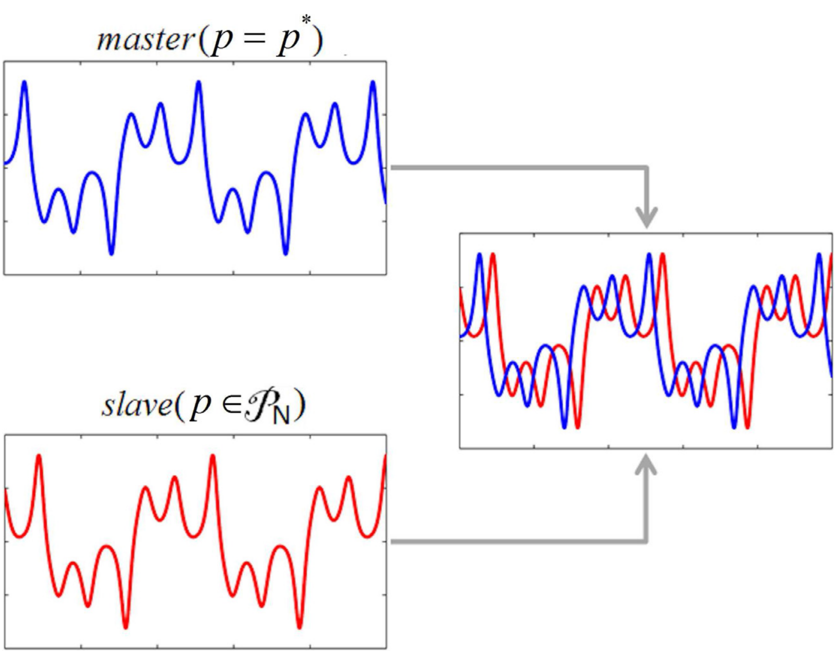


**Fig. 1.** PSLS algorithm, a sketch.

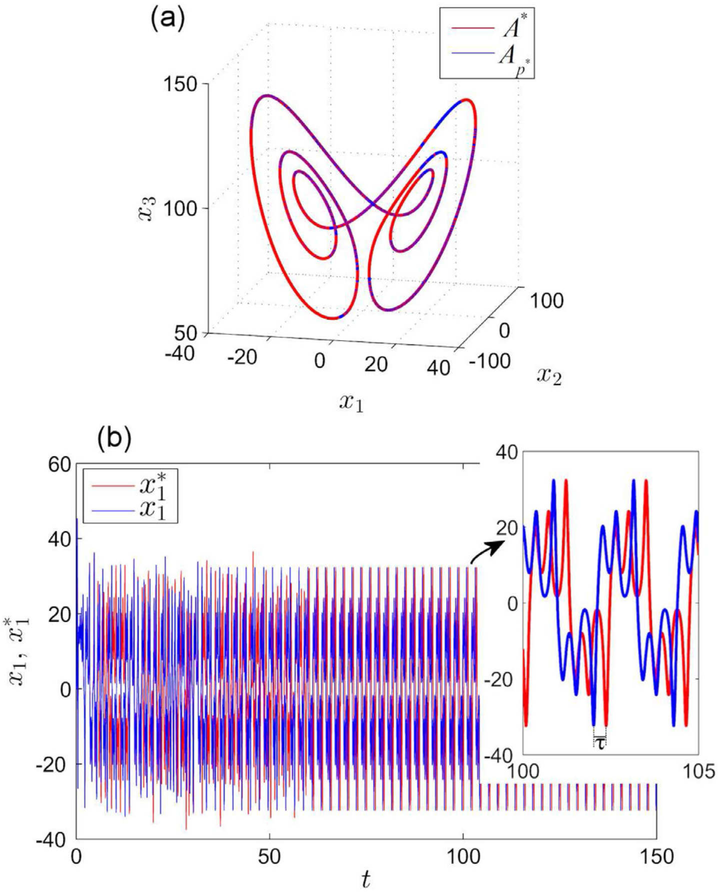


**Fig. 2.** PSLS for Lorenz system, for $p^* = 93$, using the scheme $[1p_1, 1p_2]$ with $p_1 = 90$ and $p_2 = 96$. (a) Phase overplots of the synchronized cycles. (b) Time series overplots of the first components $x_1$ and $x_1^*$ revealing the lag $\tau$ between the two cycles.