Applied Mathematics and Computation 313 (2017) 103–118

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Optimal investment–consumption strategy with liability and regime switching model under Value-at-Risk constraint

Fengxia Hu [a,*], Rongming Wang [b]

[a] School of Statistics and Mathematics, Shanghai Lixin University of Accounting and Finance, Shanghai 201209, PR China
[b] Faculty of Economics and Management, East China Normal University, Shanghai 200241, PR China

ARTICLE INFO

MSC:
60J27
91G10

Keywords:
Regime-switching model
Investment–consumption
Asset-liability management
Hamilton Jacobi Bellman equation
Value-at-Risk

ABSTRACT

This paper considers the optimal investment–consumption problem with liability subject to a maximum Value-at-Risk (denoted by *MVaR*) constraint. The model also contains regime-switching market modes, whose states are interpreted as the states of the economy. In each state of economy state, we constrain a *VaR* value for the portfolio in a short time duration, and *MVaR* is defined as the maximum value of the *VaR*s in all economy states. We suppose that both the price dynamics of the risky asset and the liability value process are governed by a Markov-modulated geometric Brownian motion. With the objective of maximizing the discounted utility of consumption, we obtain a system of HJB equations corresponding to the economy states by using the dynamic programming principle. Then, by adopting the techniques of Chen et al. (2010), we get explicit expressions of value functions. Moreover, with the help of Lagrange multiplier method, we derive the optimal investment and the optimal consumption. Finally, a numerical example is investigated, and the effects of many parameters on the optimal investment, on the optimal consumption and on *VaR* value are studied. Furthermore, we also explore how *VaR* value affects the optimal investment and the optimal consumption.

1. Introduction

Portfolio selection problem has both theoretical and practical significance in finance. The problem is to search the best allocation of wealth among some kinds of securities. Sometimes, liability, surplus, and consumption are also considered, then portfolio selection problem becomes asset-liability management problem. The pioneering contribution of portfolio selection problem is provided by Merton [17,18], who studied the optimal portfolio problem in a continuous-time framework. Afterwards, many researchers extended Mertons' model. [19] considered an optimal investment and consumption problem with the bankruptcy constraint for the asset price dynamics. Boyle and Yang [4] considered the optimal asset allocation problem in the presence of non-stationary asset returns and transaction costs. They concentrated on the multi-factor stochastic interest model and adopted a viscosity solution approach to deal with the problem. Lim and Zhou [15] proposed the portfolio selection problem with random parameters, under the help of linear–quadratic theory and backward stochastic differential equations, they solve the problem successfully.

Recently, regime switching models have received more and more attention among researchers. The main characterization of such models is that some parameters, such as interest rate of bank bond, the appreciation rate and volatility rate of

* Corresponding author.
E-mail addresses: fxhu0228@126.com (F. Hu), rmwang@stat.ecnu.edu.cn (R. Wang).

http://dx.doi.org/10.1016/j.amc.2017.04.034

risky asset, are modulated by the financial market state which is often modeled by a Markov chain. The model has a wide range of applications in economics and finance. Some papers with regime-switching models in finance include: [7,11] for asset allocation, [22,5,10,20] for ALM analysis, [8,9] for consumption–investment problems. Guo and Elliott et al. [13,14] for option pricing under incomplete markets. Usually, investment–consumption problems focus on maximizing utility from consumption. The investor would like to maximize his/her total discounted utility over the time period in which he/she invests. For example, [1] studied maximal value principle of stochastic differential equation. Lioui [16] discussed dynamic asset allocation problems in consistent time and inconsistent time, and he computed efficient costs based on mean-variance. Assa [2] investigated optimal reinsurance policy under distorted risk measure.

In this paper, we consider the optimal investment–consumption problem with liability subject to a maximum Value-at-Risk constraint. In each state of regime switching model, we constrain a *VaR* value for the portfolio in a short time duration. Maximum Value-at-Risk (denoted by *MVaR*) is defined as the maximum value of the *VaR*s in all economy states. In risk managemeng, many papers contain *VaR* constraints, for example [3]. Our idea is generated on basis of [21] where the authors only studied the total wealth, but not considered the liability and surplus separately. We suppose that the price dynamics of the risky asset and liability value are governed by a Markov-modulated geometric Brownian motion. Moreover, we assume that all market parameters, such as interest rate of a bank account, the appreciation rates of the risky asset and the liability, the volatilities of the risky asset and the liability, switch over time according the regime switching model. With the objective of maximizing the discounted utility of consumption, we obtain a system of HJB equations corresponding to the economy states by using the dynamic programming principle. Then, by adopting the techniques of [6], we get explicit expressions of value functions. Moreover, with the help of Lagrange multiplier method, we derive the optimal investment and the optimal consumption. Finally, a numerical example is investigated, and the effects of many parameters on the optimal investment, on the optimal consumption and on *VaR* value are studied. Furthermore, we also explore how *VaR* value affects the optimal investment and the optimal consumption.

The rest of this paper is organized as follows. In Section 2, we formulate the investment–consumption model studied in this paper. Section 3 derives the regime-switching HJB equation, and introduces the conditions and the method to get optimal solutions. In Section 4, we make numerical experiments to investigate the effect of many parameters on the optimal investment, on the optimal consumption and on *VaR* value are studied. Furthermore, we also explore how *VaR* value affects the optimal investment and the optimal consumption. Finally, we conclude the paper in the last section.

2. Model formulation

To begin with, we point out that all random variables in this paper are defined on a complete probability space $(\Omega, \mathfrak{F}, \mathbb{P})$. We consider a financial market in which two assets are traded continuously within the time horizon $\mathcal{T} = [0, T]$, one is the bank bond B, the other is risky asset P. $\alpha = \{\alpha(t), t \in \mathcal{T}\}$ is a continuous-time stationary Markov chain taking value in a finite state space $\chi = \{\alpha_1, \alpha_2, \dots, \alpha_N\}$. The state of χ are interpreted as different states of an economy. Following [12], we represent the state χ as a finite set of unit vectors $\varepsilon = \{e_1, e_2, \dots, e_N\}$, where e_i represents an N-dimensional column vector, and its ith element is 1, the others are 0. This is called the canonical representation of the state space of χ. The chain has a generator $Q = (q_{ij})_{N\times N}$ and stationary transition probabilities

$$p_{ij}(t) = \mathbb{P}(\alpha(t) = \alpha_j | \alpha(0) = \alpha_i), \qquad i, j = 1, 2, \dots, N.$$

With the canonical representation of the state space of χ, Elliott et al. [12] provide the following semi-martingale decomposition for α:

$$\alpha(t) = \alpha(0) + \int_0^t Q\alpha(s)ds + M(t), \tag{2.1}$$

where $\{M(t)\}_{t\in\mathcal{T}}$ is an $\mathbb{R}^N$-valued martingale with respect to the filtration $\mathfrak{F}^{\alpha}(t)$ generated by α.

Suppose the instantaneous market interest rate $r(t)$ of the bank account B is

$$r(t) = r(t, \alpha(t)) = \langle \mathbf{r}, \alpha(t)\rangle,$$

where $\langle \cdot, \cdot\rangle$ denotes an inner product in $\mathbb{R}^N$, and $\mathbf{r} = (r_1, r_2, \dots, r_N)' \in \mathbb{R}^N$ with $r_i > 0 (i = 1, 2, \dots, N)$. The corresponding price process of the bank account B is governed by

$$dB(t) = r(t)B(t)dt. \tag{2.2}$$

Suppose $\mu(t)$ and $\sigma(t)$ denote the appreciation rate and the volatility rate of the risky asset at time t, respectively. We suppose that

$$\mu(t) = \mu(t, \alpha(t)) = \langle \mu, \alpha(t)\rangle,$$

$$\sigma(t) = \sigma(t, \alpha(t)) = \langle \sigma, \alpha(t)\rangle,$$

where $\mu = (\mu_1, \mu_2, \dots, \mu_N)' \in \mathbb{R}^N$ and $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_N)' \in \mathbb{R}^N$ with $\mu_i > r_i$ and $\sigma_i > 0$, for each $i = 1, 2, \dots, N$.