Applied Mathematics and Computation 313 (2017) 209–221

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A non-stationary combined subdivision scheme generating exponential polynomials

Hongchan Zheng[a], Baoxing Zhang[a,*]

Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an 710072, Shaanxi , PR China

ARTICLE INFO

Keywords:
Non-stationary
Combined subdivision
Convergence
Exponential polynomial
generation/reproduction

ABSTRACT

In this paper, a non-stationary combined subdivision scheme is presented, which can unify several existing non-stationary approximating and interpolatory subdivision schemes. This scheme is obtained by generalizing the connection between the approximating and interpolatory schemes in the stationary case, first formalized by Maillot & Stam using a push-back operator, to the non-stationary case. For such a combined scheme, we investigate its C^l convergence and exponential polynomial generation/reproduction property and get that it can reach C^4 degree of smoothness and generate/reproduce certain exponential polynomials with suitable choices of the parameters. Besides, we give a more generalized combined scheme for the purpose of generating and reproducing more general exponential polynomials. The performance of our new schemes is illustrated by several numerical examples.

1. Introduction

Subdivision schemes are efficient tools to generate smooth curves and surfaces and they play an important role in computer graphics and wavelets. In general, subdivision schemes can be divided into approximating and interpolatory ones. These two kinds of schemes are related by a deep connection, which is fist formalized by Maillot & Stam [1] using a push-back operator to transform an approximating polyline at each subdivision step to an interpolatory polyline. Since then, there have been numerous works on the generation of interpolatory subdivision schemes from the approximating ones by using this kind of connection. Novora & Romani [2] constructed a ternary 4-point combined scheme which can unify quite a number of existing approximating and interpolatory schemes using this kind of connection. Luo & Qi [3] analyzed the interpolatory schemes obtained from the approximating ones by the push-back operation systematically in the univariate case. Lin et.al. [4] applied the push-back operation to the surface subdivision schemes and derived surface interpolatory schemes and combined ones. Besides, Zhang & Wang [5] proposed a kind of semi-stationary subdivision schemes by the push-back operator. For other references on the generation of interpolatory schemes from the approximating ones, refer to [6–8] and references therein.

The process of obtaining interpolatory subdivision schemes using the push-back operation in the above works is easy to manipulate and can also be used to design new subdivision schemes. However, all these works are restricted to the stationary schemes which can only generate/reproduce algebraic polynomials. As is known, non-stationary subdivision schemes can generate richer function spaces and reproduce conic sections, astroid and cardioid etc, which can not be done in the

* Corresponding author.
E-mail addresses: zhenghc@nwpu.edu.cn (H. Zheng), baoxingzhang@yeah.net (B. Zhang).

http://dx.doi.org/10.1016/j.amc.2017.05.066

stationary case. Therefore, in this paper, we generalize the push-back operation to the non-stationary case and construct a non-stationary combined approximating and interpolatory subdivision scheme. This new scheme is derived starting from the cubic exponential B-spline scheme. We show that our non-stationary combined scheme can unify several existing non-stationary approximating and interpolatory schemes. For such a combined scheme, we investigate its C^l convergence and exponential polynomial generation/reproduction property and show that it can reach C^4 smoothness and generate/reproduce certain exponential polynomials with suitable choices of the parameters. Moreover, we give a more generalized combined scheme for the purpose of generating and reproducing more general exponential polynomials. We also provide some numerical examples to show the performance of our new schemes.

The rest of the paper is organized as follows. In Section 2, we recall some basic knowledge about subdivision schemes. The new non-stationary combined subdivision scheme introduced in this paper is given in Section 3 while its C^l convergence and exponential polynomial generation/reproduction property is investigated in Sections 4 and 5, respectively. In Section 6, we move a further step and give a more generalized version of our new combined scheme.

2. Background

In this section, we recall some definitions and known facts which form the basis of the rest of this paper. Let $l(\mathbb{Z})$ denote the linear space of real sequences. Given a sequence of initial control points $\boldsymbol{q}^0 = \{q_j^0, j \in \mathbb{Z}\} \in l(\mathbb{Z})$, we consider the univariate non-stationary binary subdivision scheme

$$(\boldsymbol{q}^{k+1})_i = (S_{\boldsymbol{a}^k}\boldsymbol{q}^k)_i := \sum_j a_{i-2j}^k q_j^k, \qquad k \geq 0,$$

where $S_{\boldsymbol{a}^k}$ is the k-level subdivision operator mapping $l(\mathbb{Z})$ to $l(\mathbb{Z})$, $\boldsymbol{a}^k = \{a_i^k, i \in \mathbb{Z}\}$ is the k-level mask with finite length and, we denote this subdivision scheme by $\{S_{\boldsymbol{a}^k}\}_{k\geq 0}$. The k-level symbol of the mask $\boldsymbol{a}^k$ is the Laurent polynomial $a^k(z) = \sum_{i\in\mathbb{Z}} a_i^k z^i$. If the subdivision rules do not depend on the refinement level, i.e. $\boldsymbol{a}^k = \boldsymbol{a}$ for $k \geq 0$, the subdivision scheme becomes a stationary one and we denote it by $S_{\boldsymbol{a}}$ or simply S for short.

By attaching q_i^k to $i/2^k$ for $i \in \mathbb{Z}$, $k \geq 0$, we say the non-stationary subdivision scheme $\{S_{\boldsymbol{a}^k}\}_{k\geq 0}$ converges to a function $f_{\boldsymbol{q}^0} \in C^0$, the linear space of continuous functions, for the bounded initial control sequence $\boldsymbol{q}^0$, if

$$\lim_{k\to\infty} ||f_{\boldsymbol{q}^0}\left(\frac{i}{2^k}\right) - \boldsymbol{q}^k||_\infty = 0.$$

In this case, we say the scheme $\{S_{\boldsymbol{a}^k}\}_{k\geq 0}$ is C^0 convergent. If $f_{\boldsymbol{q}^0} \in C^l$, $\{S_{\boldsymbol{a}^k}\}_{k\geq 0}$ is said to be C^l convergent.

Recall that a subdivision scheme is an interpolatory one if the corresponding masks $\{\boldsymbol{a}^k\}_{k\,\geq\,0}$ satisfy

$$a_{2i}^k = \delta_{i,0}.$$

This condition guarantees that all points generated at a given level k will be kept in the next level $k+1$.

The definition of exponential polynomial spaces can be reviewed as follows.

Definition 1. ([8]) Let $T \in \mathbb{Z}_+$ and $\boldsymbol{\gamma} = \{\gamma_0, \gamma_1, \ldots, \gamma_T\}$ with $\gamma_T \neq 0$ a finite set of real or imaginary numbers. Let D^n be the nth order differential operator. The space of exponential polynomials $V_{T,\,\boldsymbol{\gamma}}$ is defined as

$$V_{T,\boldsymbol{\gamma}} := \{f : \mathbb{R} \to \mathbb{C}, f \in C^T(\mathbb{R}) : \sum_{j=0}^{T} \gamma_j D^j f = 0\}.$$

The exponential polynomial space $V_{T,\,\boldsymbol{\gamma}}$ can be characterized by the following lemma.

Lemma 1. *([8]) Let $\gamma(z) = \sum_{j=0}^{T} \gamma_j z^j$ and denote by $\{\theta_l, \tau_l\}_{l=1,\ldots,N}$ the set of zeros with multiplicity satisfying*

$$\gamma^{(r)}(\theta_l) = 0, \quad r = 0, \ldots, \tau_l - 1, \quad l = 1, \ldots, N.$$

Then

$$T = \sum_{l=1}^{N} \tau_l, \quad V_{T,\boldsymbol{\gamma}} := span\{x^r e^{\theta_l x}, r = 0, \ldots, \tau_l - 1, l = 1, \ldots, N\}.$$

3. The non-stationary combined subdivision scheme

The main purpose of this section is to give the new non-stationary combined approximating and interpolatory subdivision scheme. Before that, let us first briefly review the connection between the cubic B-spline scheme and the D-D 4-point interpolatory scheme as follows [4].