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Global well-posedness of two-dimensional magneto-micropolar equations with partial dissipation

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ABSTRACT

This paper aims at the global regularity of solutions to the two-dimensional (2D) incompressible magneto-micropolar equations with partial dissipation. We are able to deal with two partial dissipation cases and establish the global regularity of solutions for each case.

1. Introduction

The 3D incompressible magneto-micropolar fluid equations can be written as

$$\begin{cases} \partial_t u + u\cdot\nabla u = (\mu+\chi)\Delta u - \nabla\pi + b\cdot\nabla b + 2\chi\nabla\times\omega, \\ \partial_t\omega + u\cdot\nabla\omega - \alpha\nabla\nabla\cdot\omega + 2\chi\Omega = \kappa\Delta\omega + 2\chi\nabla\times u, \\ \partial_t b + u\cdot\nabla b = \nu\Delta b + b\cdot\nabla u, \\ \nabla\cdot u = 0,\ \ \nabla\cdot b = 0, \\ (u,\omega,b)(x,y,z,0) = (u_0,\omega_0,b_0)(x,y,z), \end{cases} \tag{1.1}$$

where $(x,y,z)\in\mathbb{R}^3$ and $t\ge 0$, $u(x,y,z,t)$, $\omega(x,y,z,t)$, $b(x,y,z,t)$ and $\pi(x,y,z,t)$ denote the velocity of the fluid, micro-rotational velocity, the magnetic field and the hydrostatic pressure respectively. μ, χ and $\frac{1}{\nu}$ are, respectively, kinematic viscosity, vortex viscosity and magnetic Reynolds number. κ and α are angular viscosities. By setting

$$u = (u_1(x,y,t), u_2(x,y,t), 0),\ \ \omega = (0,0,\omega(x,y,t)),\ \ b = (b_1(x,y,t), b_2(x,y,t), 0),$$

the 3D magneto-micropolar equations reduce to the 2D magneto-micropolar equations

$$\begin{cases} \partial_t u + u\cdot\nabla u = (\mu+\chi)\Delta u - \nabla\pi + b\cdot\nabla b + 2\chi\nabla\times\omega, \\ \partial_t\omega + u\cdot\nabla\omega + 2\chi\Omega = \kappa\Delta\omega + 2\chi\nabla\times u, \\ \partial_t b + u\cdot\nabla b = \nu\Delta b + b\cdot\nabla u, \\ \nabla\cdot u = 0,\ \ \nabla\cdot b = 0, \\ (u,\omega,b)(x,y,0) = (u_0,\omega_0,b_0)(x,y). \end{cases} \tag{1.2}$$

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The magneto-micropolar fluid equations (1.2) was considered in [1,9,10], where (1.2) was used to describe the motion of an incompressible, electrically conducting micropolar fluid in the presence of an arbitrary magnetic field. A generalization of the 2D magneto-micropolar equations (1.2) is given by

$$\begin{cases} \partial_t u_1 + (u\cdot\nabla)u_1 + \partial_x\pi = \mu_{11}\partial_{xx}u_1 + \mu_{12}\partial_{yy}u_1 + (b\cdot\nabla)b_1 + 2\chi\,\partial_y\omega, \\ \partial_t u_2 + (u\cdot\nabla)u_2 + \partial_y\pi = \mu_{21}\partial_{xx}u_2 + \mu_{22}\partial_{yy}u_2 + (b\cdot\nabla)b_2 - 2\chi\,\partial_x\omega, \\ \partial_t\omega + (u\cdot\nabla)\omega + 2\chi\omega = \kappa_1\partial_{xx}\omega + \kappa_2\partial_{yy}\omega + 2\chi\Omega, \\ \partial_t b_1 + (u\cdot\nabla)b_1 = \nu_{11}\partial_{xx}b_1 + \nu_{12}\partial_{yy}b_1 + b\cdot\nabla u_1, \\ \partial_t b_2 + (u\cdot\nabla)b_2 = \nu_{21}\partial_{xx}b_2 + \nu_{22}\partial_{yy}b_2 + b\cdot\nabla u_2, \\ \nabla\cdot u = 0,\ \ \nabla\cdot b = 0, \\ (u,\omega,b)(x,y,0) = (u_0,\omega_0,b_0)(x,y), \end{cases} \tag{1.3}$$

where $\Omega = \nabla\times u = \partial_x u_2 - \partial_y u_1$. Clearly, if

$$\mu_{11} = \mu_{12} = \mu_{21} = \mu_{22} = \mu + \chi,\ \ \kappa_1 = \kappa_2 = \kappa,\ \ \nu_{11} = \nu_{12} = \nu_{21} = \nu_{22} = \nu,$$

then (1.3) reduces to the standard 2D magneto-micropolar equations in (1.2). This generalization is capable of modeling the motion of anisotropic fluids for which the diffusion properties in different directions are different. In fact, in certain physical regimes and under suitable scaling, certain components of the dissipation can become small and be ignored, as in the case of Prandtl's boundary layer equations. The horizontal velocity equation of Prandtl's boundary layer theory only contains the vertical dissipation (no horizontal dissipation). It may be possible in certain physical circumstances the velocity equation concerned here has only partial viscosity dissipation. In addition, mathematically, (1.3) allows us to explore the smoothing effects of various partial dissipations.

The magneto-micropolar equations play an important role in engineering and physics, and the mathematical study of this equations has attracted many mathematicians (see, e.g., [2,14–19,22]). When (1.2) has full dissipation (namely, μ, χ, κ, $\nu > 0$), the global existence and uniqueness of solutions was obtained in [15,18]. In the case of inviscid magneto-micropolar equations, the global regularity problem is still a challenging open problem. Therefore, it is natural to study the intermediate cases, namely, (1.3) with partial dissipation. Due to the complex structure of (1.3), when there is only partial dissipation, the global regularity problem can be quite difficult. However, several important progresses have recently been made on this direction. In [22], Yamazaki obtained the global regularity of (1.3) with zero angular viscosity (namely, $\kappa_1 = \kappa_2 = 0$ and other coefficients being positive). In [2], the global well-posedness of (1.3) with mixed partial dissipation (namely, $\mu_{11} = \mu_{21} = \nu_{12} = \nu_{22} = \kappa_2 = 0$ and $\mu_{12} = \mu_{22} = \nu_{11} = \nu_{21} = \kappa_1 = 1$ or $\mu_{11} = \mu_{21} = \nu_{12} = \nu_{22} = \kappa_2 = 1$ and $\mu_{12} = \mu_{22} = \nu_{11} = \nu_{21} = \kappa_1 = 0$) was obtained by Cheng and Liu. Very recently, Regmi and Wu [16] studied (1.3) with $\mu_{11} = \mu_{22} = \nu_{12} = \nu_{22} = \kappa_2 = 0$ and $\mu_{12} = \mu_{21} = \nu_{11} = \nu_{21} = \kappa_1 = 1$ or $\mu_{11} = \mu_{12} = \nu_{11} = \nu_{21} = \kappa_1 = 1$ and $\mu_{21} = \mu_{22} = \nu_{12} = \nu_{22} = \kappa_2 = 0$, and obtained the global regularity of solutions of (1.3) with these partial dissipations.

When $\omega = 0$ or $b = 0$, the 2D magneto-micropolar equations (1.2) reduces to the 2D magneto-hydrodynamic equations or 2D micropolar equations, respectively. Quite a few important global regularity results are available for these two equations with partial dissipation, see, e.g., [3–8,11–13,20,21].

The focus of this paper will be on the global well-posedness problem on 2D magneto-micropolar equations (1.3) with partial dissipation. We deal with two partial dissipation cases and establish the global regularity for each case. The main results are as follows. Without loss of generality, we set $\chi = \frac{1}{2}$ in the rest of the paper.

The first case is to set $\mu_{11} = \mu_{22} = \nu_{21} = \nu_{22} = \kappa_1 = 0$ and $\mu_{12} = \mu_{21} = \nu_{11} = \nu_{12} = \kappa_2 = 1$. Therefore, (1.3) reduces to

$$\begin{cases} \partial_t u_1 + (u\cdot\nabla)u_1 + \partial_x\pi = \partial_{yy}u_1 + (b\cdot\nabla)b_1 + \partial_y\omega, \\ \partial_t u_2 + (u\cdot\nabla)u_2 + \partial_y\pi = \partial_{xx}u_2 + (b\cdot\nabla)b_2 - \partial_x\omega, \\ \partial_t\omega + (u\cdot\nabla)\omega + \omega = \partial_{yy}\omega + \Omega, \\ \partial_t b_1 + (u\cdot\nabla)b_1 = \partial_{xx}b_1 + \partial_{yy}b_1 + b\cdot\nabla u_1, \\ \partial_t b_2 + (u\cdot\nabla)b_2 = b\cdot\nabla u_2 \\ \nabla\cdot u = 0,\ \ \nabla\cdot b = 0, \\ (u,\omega,b)(x,y,0) = (u_0,\omega_0,b_0)(x,y). \end{cases} \tag{1.4}$$

Theorem 1.1. *Assume $(u_0,\omega_0,b_0) \in H^2(\mathbb{R}^2)$, and $\nabla\cdot u_0 = \nabla\cdot b_0 = 0$. Then the 2D magneto-micropolar equations (1.4) has a unique global classical solution (u, ω, b) satisfying, for any $T > 0$,*

$$(u,\omega,b) \in L^\infty([0,T]; H^2(\mathbb{R}^2)).$$