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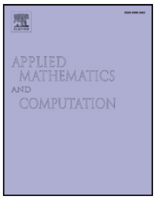

Environment promotes the evolution of cooperation in spatial voluntary prisoner's dilemma game

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ABSTRACT

In reality fitness can be affected by the environment. We explore the evolution of cooperation with the influence of environment on prisoners' dilemma game with voluntary participation. An individual's fitness is redefined to involve one's own payoff and the average performance of neighbors via preference level u. When u equals zero, the game falls back to its traditional form in which the fitness of an individual simply reflects one's own benefit. When u is larger than 0, the environment plays a role. Numerical simulations show that, for small b, increasing u enables the frequency of cooperation to increase monotonously and even dominate the whole population. For large b, although cooperators are exploited by defectors, the existence of loners protects them from getting wiped out. Finally three strategies start to exhibit cyclic dominance.

1. Introduction

Cooperation is ubiquitous in nature and societies. The health of our body relies on mutual cooperation of different cells. In bee colonies, the workers give up their reproductive opportunities and help the queen to reproduce. Despite the wide presence of cooperative social behavior, it is still difficult to understand the emergence and maintenance of cooperation, because cooperators should be eliminated in the process of evolution according to natural selection [1–3]. Targeting this issue, evolutionary game theory has provided a theoretical framework to study puzzling dilemmas and stimulated many studies ranging from evolutionary biology to behavioral science to statistical physics [4–8]. In particular, the prisoner's dilemma game (PDG), serving as a paradigm for unfavorably structured pairwise interactions, has attracted considerable attention to explore the evolution of cooperation in both theoretical and experimental studies [6,9–15]. In this simple game, two players must synchronously choose to either cooperate or defect without knowing the choice of the opponent. Both players receive reward R for mutual cooperation and punishment P for mutual defection. If one cooperator encounters one defector, the defector gets the temptation to defect T, while the cooperator receives a sucker's payoff S. The ranking of these four payoffs strictly satisfies $T > R > S > P$ and $2R > T+S$. Rationally, defection always represents the best choice irrespective of the opponent's decision, which leads to a deadlock situation of mutual defection.

Over the past decades, many mechanisms have been proposed to offset the above unfavorable outcome and to enhance the cooperativeness of populations [16–18]. Nowak [19] summarized five mechanisms: kin selection [20], direct reciprocity

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[21], indirect reciprocity [22], network reciprocity [23] and group selection [19]. Among these achievements, spatial structure has proven to be an effective way to promote the evolution of cooperation. These works sparked the study of a variety of topologies such as ER graphs [24,6–12], BA scale-free networks [25], and interdependent networks [26]. Meanwhile, different factors have also been considered in structured population for exploring its impact on the evolution of cooperation including reputation [27], extortion [28,29], coevolution [30,31], asymmetry [32], age structure [33], social diversity [34,35], to name a few. Besides, voluntary participation [36], serving as a risk averse strategy, has proven to be an efficient way for maintaining cooperation [37]. In its basic form, players are afforded a third option —loner (L). The risk averse loners prefer receiving small fixed income to participating in the PD game, thus avoiding the fully defecting state by reverting to a cyclic dominance state of three strategies [38,39].

Environment plays an indispensable role in our daily life and may exert a positive impact on the evolution of cooperation [40–43]. Thus an interesting question appears: if we combine the environmental factor and voluntary participation with structured population in PD game to explore the evolution of cooperation, can this setup promote cooperation? We use Monte Carlo simulations to answer this question, finding that cooperation can be greatly enhanced. Especially, the larger the role of the environment, the higher the level of cooperation.

The rest of this paper is composed of three sections. In Section 2, we present our evolutionary game model, including the new definition of fitness. Section 3 gives a description of numerical simulation results. Finally, we discuss the results and conclude the paper in Section 4.

2. Model

Let’s first consider the interaction network, where each player occupies the nodes of $L*L$ square lattice with periodic boundary conditions. Each player is set to act as either a cooperator, a defector or a loner with equal probability, which can be described as

$$S_x = (1,0,0)^T, S_y = (0,1,0)^T \text{ or } S_z = (0,0,1)^T. \tag{1}$$

The risk averse loners and their opponents always obtain the small but fixed profit σ. Following the common practice [38,39], we fix $\sigma = 0.3$ and choose the weak PD game, in which the payoffs are defined as $R = 1$, $P = S = 0$, and $T = b > 1$. Thus the payoff matrix can be expressed by matrix M.

$$M = \begin{pmatrix} 1 & 0 & \sigma \\ b & 0 & \sigma \\ \sigma & \sigma & \sigma \end{pmatrix} \tag{2}$$

At each time step, player i plays the game with its nearest neighbors and obtains its incomes P_i:

$$P_i = \sum_{j \in N_i} S_i^T M S_j^T, \tag{3}$$

where N_i represents the set of neighbors of individual$i$. The payoff P_j of neighbors can be obtained in the same way.

As for the environmental factor, following Ref. [41],

$$E_i = \sum_{l=1}^{N_i} \left(\frac{e^{p_l}}{\sum_{j=1}^{N_i} e^{P_j}} P_l \right), \tag{4}$$

where N_i represents the neighbors of focal individual i, P_l (P_j) is the payoff of player $l(j)$ who is one of the neighbors of player$i$. Then, we define the fitness of player i according to the following expression:

$$\begin{array}{ll} F_i = (1-u)P_i + uE_i & i \text{ is } C \text{ or } D, \\ F_i = P_i & i \text{ is } L, \end{array} \tag{5}$$

where u represents the contribution of the environment in the calculation of an individual’s fitness. When $u = 0$, the game falls back to its traditional form, which does not consider the influence of the environment, while $u = 1$ means that individuals’ fitness relies on the environment completely. Here, we discuss the value of u ranging from 0 to 0.5.

The game is implemented using Monte Carlo simulations (MCS). First, player i and j are randomly selected and their fitness evaluated according to Eq. (5). Then, player i adopts the strategy from neighbor j with the probability W depending on the fitness difference:

$$\mathrm{W} = \frac{1}{1 + \exp\left[\left(F_i - F_j \right) / K \right]}, \tag{6}$$

where K denotes the noise of selection, including irrationality and errors [44,45]. Because the effect of noise K has been well studied in previous papers [46,47], we set K to 0.1. During a full Monte Carlo step all players update their strategies. All simulations were carried out on lattices with $L = 400$.