Applied Mathematics and Computation 315 (2017) 96–112

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Convergence of strong time-consistent payment schemes in dynamic games

Leon Petrosyan[a], Artem Sedakov[a], Hao Sun[b], Genjiu Xu[b,*]

[a] Saint Petersburg State University, 7/9 Universitetskaya nab., Saint Petersburg 199034, Russia
[b] Department of Applied Mathematics, Northwestern Polytechnical University, Xian, Shaanxi 710072, PR China

ARTICLE INFO

MSC:
91A12
91A25
15A04

Keywords:
Game theory
Dynamic games
Linear transformation
Cooperation
Strong time consistency

ABSTRACT

The problem of consistency of a solution over time remains an important issue in cooperative dynamic games. Payoffs to players prescribed by an inconsistent solution may not be achievable since such a solution is extremely sensitive to its revision in the course of a game developing along an agreed upon cooperative behavior. The paper proposes a strong time-consistent payment scheme which is stable to a revision of cooperative set solutions, e.g., the core. Using a linear transformation of the solution, it becomes possible to obtain non-negative payments to players. In the paper, we also deal with a limit linear transformation of the solution whose convergence is proved. Developing a non-negative strong time-consistent payment scheme in a closed form, we guarantee that the solution supported by the scheme will not be revised over time.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction

When studying a cooperative game, it is often supposed that worths of all possible coalitions are determined by the characteristic function, i.e., are given exogenously. In the present research, we assume that players are initially involved in a strategic non-cooperative conflict modeled by a dynamic game. Based on its strategic setting, we construct a cooperative version of the game and call it a cooperative dynamic game. Since the game is initially defined in strategic form, the worths of all possible coalitions should be determined by players themselves. In the paper, we determine the worth of a coalition in the sense of Von Neumann and Morgenstern [23] as the maximum value that the coalition can guarantee itself in the worst-case scenario when its complement plays against the coalition. It is worth noting that the worth of the coalition may be determined in other ways by a Nash equilibrium [2,15].

Having determined the worths of coalitions, and therefore, the characteristic function, one may propose a solution of the cooperative dynamic game. However, in this case we should keep in mind the *consistency* of the solution which means that it will not be negotiable and revised in the course of the game developing along the agreed upon cooperative behavior. In cooperative games consistency is an important characteristic for an axiomatic formulation of solutions. Davis and Maschler [4] first introduced reduced game consistency for characterizing the kernel of cooperative games. Sobolev [17], Hart and Mas-Colell [10] then studied reduced game consistency for the Shapley value. The property of reduced game consistency has been investigated for various solution concepts. Most of these results can be found in the survey papers by

* Corresponding author.
E-mail addresses: l.petrosyan@spbu.ru (L. Petrosyan), a.sedakov@spbu.ru (A. Sedakov), hsun@nwpu.edu.cn (H. Sun), xugenjiu@nwpu.edu.cn, g.xu@foxmail.com (G. Xu).

http://dx.doi.org/10.1016/j.amc.2017.07.030
0096-3003/© 2017 Elsevier Inc. All rights reserved.

Driessen [5] and Thomson [18]. Hamiache [8] presented a consistency axiom named "associated consistency" to characterize the Shapley value. According to associated consistency, the solution is invariant under the adaptation of the game into an associated game. The consistency axiom is defined with reference to an associated game rule, i.e., a linear transformation in the cooperative game space. A sequence of repeated associated games is also constructed by Hamiache in terms of the linear transformation. The matrix approach was applied to study the convergence of the sequence and also the characterizations for the Shapley value by Hamiache [9] and Xu et al. [24,25].

In dynamic games, it is known that most cooperative solutions, e.g., the Shapley value, the core, the τ-value, the *CIS*-value are inconsistent in general, but they become consistent under additional assumptions on a class of players' payoff functions [6]. If we do not suppose that the payoff functions are of a special class, players will revise the solution over time owning to its inconsistency and thus break the initial cooperative agreement. Payment schemes guaranteeing the realization of the cooperative agreement from the perspective of its consistency are widely implemented in the application of dynamic cooperative games. For example, in a dynamic cooperative game defined by an economic lot sizing problem [19], the time consistency property is one of the properties claimed from allocations. A model of collaborative environmental management is studied in [26] where a time-consistent solution is constructed with the use of a payoff distribution mechanism of a special kind. Another environmental model is considered as a cooperative dynamic game with a finite horizon in [7] where the cooperative solution of the game is "supported" with a payment scheme based on transfers. An incentive scheme using the Shapley value is developed in [3] for a cooperative dynamic model in the competitive electricity market environment with a predefined time horizon. A payoff distribution mechanism allocating a cooperative solution over time and thereby sustaining the cooperative agreement is proposed in [27] for an economy with a renewable resource and resource extractors, a model of corporate joint venture, collaborative environmental management, and a dormant-firm cartel. The application of dynamic models using an evolutionary approach can be found in [11,12,20–22]. Here, we consider set solutions of the cooperative game and, specifically, the core as a classical example of such solutions. Adopting the approach used in [13,14] for constructing a time-consistent payment scheme (not necessarily non-negative) for point solutions, we develop a more strict scheme, a strong time-consistent scheme, which is applicable for set solutions, e.g., for the core of the cooperative dynamic game. An alternative approach can be found in [16].

Our paper makes two contributions. First, it proposes a non-negative payment scheme which reallocates a core element over time; thus, ensuring players obtain their agreed upon payoffs in the game. Second, it provides a strong time-consistent payment scheme in a closed form which guarantees that the solution will not be negotiable in the course of the game. Strong time consistency is achieved by a linear transformation of the characteristic function along the cooperative trajectory. Additionally, the paper studies the convergence of the linear transformation specifying its properties.

2. The model

Let N be a finite set of players, $|N| = n \geqslant 2$, and X be a finite set of states. Consider an ℓ-stage game $\Gamma(x_1, \ell)$ over the set X which starts from the initial state $x_1 \in X$ and lasts for ℓ stages. Denote a behavior of player $i \in N$ in state $x \in X$ at stage $t \in \{1, \dots, \ell\}$ by $b_i(t, x)$, and the set of all possible behaviors of player i in x by $B_i(x)$ for any t. Let $b(t, x) = (b_1(t, x), \dots, b_n(t, x))$ and $B(x) = B_1(x) \times \cdots \times B_n(x)$. The game dynamics is given by a single-valued mapping T that any profile $(x, b(t, x))$ at stage t maps to a state from set X at the next stage $t+1$. The game can be described as follows. At the first stage in the initial state $x_1 \in X$ players choose their behaviors $b_i(1, x_1) \in B_i(x_1)$, $i \in N$, and the process moves to the next state $x_2 = T(x_1, b(1, x_1)) \in X$. In x_2 players choose their behaviors $b_i(2, x_2) \in B_i(x_2)$, $i \in N$, and the process moves to $x_3 = T(x_2, b(2, x_2)) \in X$, etc. At the last stage ℓ in a state $x_\ell \in X$ players choose their behaviors $b_i(\ell, x_\ell) \in B_i(x_\ell)$, $i \in N$, and the game ends.

Definition 2.1. A strategy u_i of player $i \in N$ is a map that assigns behavior $b_i(t, x) \in B_i(x)$ to each state $x \in X$ and stage $t \in \{1, \dots, \ell\}$, i.e., $u_i(t, x) = b_i(t, x)$. A profile $u = (u_1, \dots, u_n)$ is a strategy profile in ℓ-stage game $\Gamma(x_1, \ell)$.

Definition 2.2. A sequence $(x_1, \dots, x_\ell)$, where $x_{t+1} = T(x_t, b(t, x_t))$, $t = 1, \dots, \ell - 1$, is called a trajectory in ℓ-stage game $\Gamma(x_1, \ell)$ corresponding to the strategy profile u such that $u(t, x_t) = b(t, x_t)$.

Note that any strategy profile uniquely defines players' behaviors in each state, and therefore the trajectory corresponding to the given strategy profile is uniquely defined.

If the strategy profile u and the corresponding trajectory $(x_1, \dots, x_\ell)$ are realized in game $\Gamma(x_1, \ell)$, players' payoffs along the trajectory (i.e., players' payoffs in the game) are defined as $\sum_{t=1}^{\ell} h_i(x_t, b(t, x_t))$, $i \in N$, where $h_i(x_t, b(t, x_t))$ is a non-negative stage payoff to player $i \in N$ in state x_t at stage t. Since the strategy profile u uniquely defines the trajectory, the payoffs to players are also uniquely determined and can be represented as functions of u:

$$K_i(x_1, u) = \sum_{t=1}^{\ell} h_i(x_t, b(t, x_t)), \quad i \in N. \tag{1}$$

Thus, $\Gamma(x_1, \ell)$ can be considered as a normal-form game. Denote the subset of set X, which consists of states reachable from the initial state x_1 in exactly t stages by $R(x_1, t)$, $t = 1, \dots, \ell - 1$. More specifically, the subset is recursively defined in the following way:

Download English Version:

https://daneshyari.com/en/article/5775635

Download Persian Version:

https://daneshyari.com/article/5775635

Daneshyari.com