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Exponential synchronization of a class of neural networks with sampled-data control

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ABSTRACT

This paper investigates the problem of the master-slave synchronization for a class of neural networks with discrete and distributed delays under sampled-data control. By introducing some new terms, a novel piecewise time-dependent Lyapunov-Krasovskii functional (LKF) is constructed to fully capture the available characteristics of real sampling information and nonlinear function vector of the system. Based on the LKF and Wirtinger-based inequality, less conservative synchronization criteria are obtained to guarantee the exponential stability of the error system, and then the slave system is synchronized with the master system. The designed sampled-data controller can be obtained by solving a set of linear matrix inequalities (LMIs), which depend on the maximum sampling period and the decay rate. The criteria are less conservative than the ones obtained in the existing works. A numerical example is presented to illustrate the effectiveness and merits of the proposed method.

1. Introduction

In the past several years, neural networks have been attracting considerable attention since it has a wide range of applications in the fields of science such as biological system, pattern recognition, fixed-point computations, combinatorial optimization and so on [1]. It is well known that the neural networks can be achieved by large scale integrated circuit. Therefore, the induced delays will be produced due to the communication of neurons and the finite switching speed of amplifiers. Time delays are the main causes of instability and poor performances of neural networks. The problem of the stability analysis for the neural networks with time delays is an important research field. For example, the stabilization of various neural networks with time delays has been broadly studied and numerous stability criteria have been obtained via the linear matrix inequalities (LMIs) method [2–7]. In [8–10], the authors devoted to study the problems of state estimation for neural networks with time-delays, where the LMIs method was used to obtain the expected state estimator gain matrices. In [11] and [12], the authors investigated the negative analysis of the delayed neural network. By using integral partitioning technique, the distributed-delay-dependent conditions were established for the cellular neural networks in [13].

On the other hand, the exponential synchronization of neural networks has been widely studied due to its applications in nonlinear oscillation synchronization, secret communication and other nonlinear fields. Many methods have been applied

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to the exponential synchronization of neural networks such as fuzzy control, time-delay feedback control, sampled-data control, and so on. With the development of high-speed of digital computing technology, the digital devices which have the advantages of low installation cost, better reliability and easy maintenance are gradually utilized in industrial applications. This allows the system only using the samples of continuous-time measurement signals at discrete time instants. These samples are used to control the continuous-time plant through a zero-order hold (ZOH). The approach drastically increases the efficiency of bandwidth usage and reduces the system information. This control system is considered to be sampled-data system. Because the control signals between any two continuous sampling instants will be held constant and only be changed at each sampling time, the analysis and synthesis of sampled-date systems are difficult and complex. Many approaches have been used to sampled-data control systems. The most popular approach is the so-called input delay approach [14–17]. In this approach, the sampled-data system is shaped into a continuous-time system with time-varying input delay induced by the ZOH. Moreover, the stability condition in accordance with LMIs will be established by various methods like time-dependent Lyapunov functional approach [18–22], discontinuous Lyapunov functional technique [23,24], and looped-functional based method [25].

As well known, discontinuous Lyapunov functional approach has been adopted to derive less conservative criteria effectively for sampled-data systems. For example, in [26,27], a new free-matrix-based time-dependent discontinuous Lyapunov functional was introduced for stability analysis of sampled-data systems. The proposed discontinuous terms utilized more information of the sawtooth structural sampling patter. In [28], the authors investigated the synchronization problem of neural networks with time-varying delay under sampled-data control by using the input delay approach. A discontinuous Lyapunov functional which took full advantage of the sawtooth structure characteristic of sampling interval was constructed based on the extended Wirtinger inequality. By construction of a suitable LKF and utilization of Finsler's lemma, novel delay-dependent criteria for the synchronization of coupled neural networks with interval time-varying delays and leakage delay were derived in terms of LMIs in [29]. Furthermore, the authors in [30] proposed a newly augmented LKF which combined with a reciprocally convex combination technique. Some improved stabilization criteria were established in term of LMIs by taking different interval of integral terms of LKF. However, the above mentioned results suffer some conservatism since the sampling pattern is not adequately considered for sampled-data control systems, which motivates us to this study.

As discussed above, in this paper, a sampled-data feedback control for master-slave synchronization of neural networks with discrete and distributed delays has been proposed. Inspired in [31], a novel LKF is constructed, which makes full use of the sawtooth structure characteristics of the sampled-data systems. Some new stability conditions are obtained to guarantee the exponential stability of error system, so that the slave system can be synchronized with the master system. By using Wirtinger-type inequality [32] and reciprocally convex method [33], the new stability conditions can be derived in a set of LMIs. Then, the sampled-data controller can be designed to guarantee the maximum sampling interval and the optimal decay rate via solving the LMIs. Results of numerical example show the effectiveness of the proposed method and the reduced conservativeness.

Notations: Throughout this paper, a real symmetric matrix $P > 0$ represents that P is a positive definite matrix, and $X > Y$ stands for $X - Y > 0$. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ is a set of $m \times n$ real matrix. diag$\{\cdots\}$ denotes the block diagonal matrix. The transpose of a matrix Q is denoted by Q^T. Moreover, for any square matrix $A \in \mathbb{R}^{n\times n}$, we define $He(A) = A + A^T$. The set of symmetric matrices of dimension n is denoted by $\mathbb{S}^n$, and a symmetric matrix by $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ * & D \end{bmatrix}$.

2. Problem statement and preliminary

Consider the neural networks with discrete and distributed delays as follows:

$$\dot{x}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t - d(t))) + D\int_{t-\tau(t)}^{t} g(x(s))ds + V(t), \tag{1}$$

where $x(t) = [x_1(t)\ x_2(t) \ldots x_n(t)]^T$ represents the state vector, $g(x(t)) = [g_1(x_1(t))\ g_2(x_2(t))\ \ldots\ g_n(x_n(t))]^T$ represents the neuron activation function, $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ are some connection weight matrices, which respectively stand for the connection weight matrix, the time-delays connection weight matrix, and the distributively delayed connection weight matrix, $C = diag\{c_1, c_2, \ldots, c_n\}$ is the positive definite diagonal matrix, $V(t) = [V_1(t)V_2(t), \ldots, V_n(t)]^T$ represents a external input vector.

The discrete delay $d(t)$ and the distributed delay $\tau(t)$ are satisfactory to the following conditions:

$$0 \le d(t) \le d, \quad \dot{d}(t) \le \mu, \tag{2}$$

and

$$0 \le \tau(t) \le \tau, \tag{3}$$

where d, μ and τ are known positive scalars.

Now, the following assumption will be used in the sequel.

Assumption 1. [31] The neuron activation function $g_i(\cdot)$ is continuous and bounded, and there exist constants F_i^- and F_i^+ such that

$$F_i^- \le \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \le F_i^+, \quad i = 1, 2, \ldots, n \tag{4}$$