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LMI approaches to input and output quantized feedback stabilization of linear systems

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ABSTRACT

This paper investigates the problem of quantized feedback stabilization for linear systems. In the controlled systems, the measurement output and control input signals are transmitted via the digital communication link, and the quantization errors are treated as sector bound uncertainties. Two different approaches to designing output feedback are developed and the corresponding design conditions in terms of solutions to linear matrix inequalities (LMIs) are presented. The resulting design is such that the homologous closed-loop system is asymptotically stable with respect to the quantization effects. Finally, we illustrate the efficiency of our main results by a numerical example.

1. Introduction

In recent years, investigating effects of signal quantization has been one of the hot research topics in the control area. For quantized systems, how to involve communication channels with limited bandwidth is interesting and has received much attention. In this setting, the transmission signals will be quantized and then coded. In order to ensure a control requirement or control performance, it is essential to design a control strategy with respect to the quantized signals. Because the quantization problem widely exists in computer-based systems, great efforts have been made to investigate the analysis and synthesis problems for systems via various quantization approaches. Elia and Mitter [1] indicated that the coarsest, or least dense, quantizer quadratically stabilizing a single input linear system is logarithmic, and which can be computed by solving a special linear quadratic regulator problem. In [2], by generalizing the result proposed in [1], the quantized feedback design problem for linear systems based on a sector bound approach has been studied. It shows that many feedback design problems with quantization are able to be converted to well-known robust problems by considering sector bound uncertainties. Because of this advantage, the sector bound approach has attracted much attention, and various results on quantization have been reported [3–12].

It should be noted that all the above works on the quantization problem are discussed for a single quantizer that is measurement output signal quantization or control input signal quantization. However, in actual remote control systems, there is a fact that the designed controller and the considered plant are physically distant, it implies that the measurement output signal which is transmitted to the designed controller and the control input signal which is transmitted to the system plant are connected through a communication network [13]. In other words, both the measurement output and the control input signals should be quantized before they are transmitted to the corresponding components of the control

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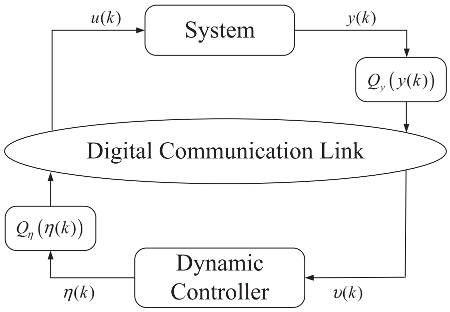

Fig. 1. Control system description.

system, respectively. So a significant issue is how to design a feedback control for a given system plant such that the close-loop system guarantees stability or a prescribed control requirement with respect to the quantization effects. To date, very few researches have discussed stabilization and control performance problems for systems with quantized input and output [14,15]. Recently, [13] has extended the result presented by [2] and studied quantized feedback systems taking into account both the quantized input and output signals by using the sector bound approach. Moreover, with no conservatism, the stabilization problem via quantized dynamic output feedback has been addressed through an auxiliary uncertain system in [13]. However, this result is difficult to be generalized directly to stabilization synthesis in the LMI framework.

Motivated by this point, this paper studies the output feedback stabilization design problem of linear systems with quantized input and output signals. Given such a quantized feedback control system, our objective is to design a control strategy such that the controlled system with the feedback controller is asymptotically stable via the LMI technique. Approaches based on linear fractional transformation representation and conventional representation for the close-loop system are developed to facilitate the control design, it is shown that the controller design conditions are portrayed in terms of LMIs. The main contributions of this paper are summarized as follows.

(1) The dynamic output feedback stabilization problem is investigated for linear systems with both input and output quantization.

(2) Two different approaches based on linear fractional transformation representation and conventional representation are proposed to deal with the output feedback stabilization design problem of linear systems with respect to the quantization effects.

(3) The designs of the output feedback controllers are given in terms of solutions to linear matrix inequalities (LMIs).

Notations: The notation $He(A)$ denotes $A + A^T$ for simplicity. In symmetric block matrices, we use the symbol (*) to represent a symmetry term.

2. Problem formulation and preliminaries

Consider a discrete-time system described by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the state variable, $u(k) \in \mathcal{R}^m$ is the control input, $y(k) \in \mathcal{R}^f$ is the measurement output; $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, and $C \in \mathcal{R}^{f\times n}$ are system matrices.

In this paper, for the controlled system (1), only the output information is assumed to be available for feedback, and the following feedback controller is given:

$$\begin{aligned} \xi(k+1) &= A_d\xi(k) + B_d\upsilon(k) \\ \eta(k) &= C_d\xi(k) + D_d\upsilon(k) \end{aligned} \tag{2}$$

where $\xi(k) \in \mathcal{R}^n$ is the state variable of the controller, $\upsilon(k) \in \mathcal{R}^f$ and $\eta(k) \in \mathcal{R}^m$ are the input and output variables of the controller, respectively. $A_d \in \mathcal{R}^{n\times n}$, $B_d \in \mathcal{R}^{n\times f}$, $C_d \in \mathcal{R}^{m\times n}$, and $D_d \in \mathcal{R}^{m\times f}$ are the controller matrices to be determined.

Now let us consider a fact that the measurement output of the system (1) and the output of the controller (2) will be quantized, respectively, before they are transmitted to the controller (2) and the system plant (1) via communication network. The framework of the control system discussed in this paper is depicted in Fig. 1. To this end, the following two logarithmic quantizers are employed:

$$\begin{aligned} \upsilon(k) = Q_y\big(y(k)\big) &= \left[Q_y^1\big(y_1(k)\big) \quad Q_y^2\big(y_2(k)\big) \quad \ldots \quad Q_y^f\big(y_f(k)\big)\right]^T \\ u(k) = Q_\eta\big(\eta(k)\big) &= \left[Q_\eta^1\big(\eta_1(k)\big) \quad Q_\eta^2\big(\eta_2(k)\big) \quad \ldots \quad Q_\eta^m\big(\eta_m(k)\big)\right]^T \end{aligned} \tag{3}$$