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Extension of some particular interpolation operators to a triangle with one curved side

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ABSTRACT

We extend some Nielson and Marshall type interpolation operators to the case of a triangle with one curved side. We study the interpolation properties of the obtained operators and of their product and Boolean sum operators, the orders of accuracy and the remainders of the corresponding interpolation formulas. Finally, we give some numerical examples.

1. Introduction

Interpolation operators for functions defined on triangles with straight sides have been constructed starting with the paper of Barnhill, Birkhoff and W. J. Gordon [3], and in many other papers as, for example, [4,6,7,9,14,23,25,26,28–31,34]. These kind of interpolation operators were applied in the piecewise generation of surfaces in computer aided geometric design (see, e.g., [2,3]) and in finite element analysis (see, e.g., [22,27,28,35]).

Recently, we have introduced some interpolation operators for functions defined on triangles with curved sides (see, e.g., [10–13,15,17,18]). They come as an extension of the interpolation operators on triangles with all straight edges. They permit essential boundary conditions to be satisfied exactly. These operators are used in finite element method for differential equation problems (Lagrange operators for Dirichlet boundary conditions, Birkhoff operators for Neumann boundary conditions and Hermite operators for Robin boundary conditions) (see, e.g., [1,8,24–26,35]) or in connection with their applications in computer aided geometric design (see, e.g., [1,2,4,6]) and in construction of surfaces which satisfy some given conditions, such as, for example, the roofs of the halls (see, e.g., [16,19–21]).

In the sequel, we consider a standard triangle, $\tilde{T}$, having the vertices $V_1 = (1, 0)$, $V_2 = (0, 1)$ and $V_3 = (0, 0)$, two straight sides Γ_1, Γ_2, along the coordinate axes, and the third side Γ_3 (opposite to the vertex V_3), which is defined by the one-to-one functions f and g, where g is the inverse of the function f, i.e., $y = f(x)$ and $x = g(y)$, with $f(0) = g(0) = 1$ (see Fig. 1). There is no restriction to consider this standard triangle $\tilde{T}$, since any triangle with one curved side can be obtained from this standard triangle by an affine transformation which preserves the form and order of accuracy of the interpolant [4].

We extend some interpolation operators introduced by Barnhill, Gregory, Marshall, Mitchell and Nielson in some papers as, for example, [4,5,28], to the case of a triangle with one curved side. We study the interpolation properties of the operators, the orders of accuracy and the remainders of the corresponding interpolation formulas using Peano's theorem (see, e.g., [33]) for functions from the Sard-type space $B_{p,q}(0, 0)$, for $p, q \in \mathbb{N}^*$ (see, e.g., [33]).

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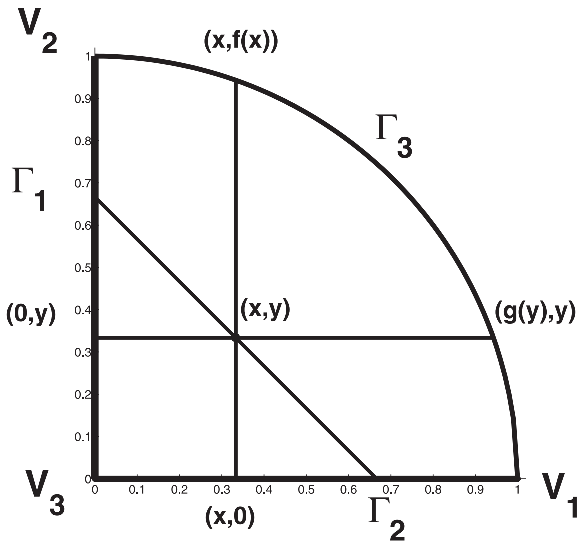

Fig. 1. Triangle $\tilde{T}$.

We also obtain some product and Boolean sum operators. We also analyze their interpolation properties, orders of accuracy and the remainders of the corresponding interpolation formulas. The last section contains some numerical examples which illustrate the obtained theoretical results.

Definition 1.1 [33]**.** The Sard-type space $B_{p,\,q}(a,\,c)$, $(p, q \in \mathbb{N},\ p+q=m)$ is the space of the functions $f : D \to \mathbb{R}$, $D = [a, b] \times [c, d]$ satisfying:

1. $f^{(p,q)} \in C(D)$;
2. $f^{(m-j,j)}(\cdot, c) \in C[a, b],\ j < q$;
3. $f^{(i,m-i)}(a, \cdot) \in C[c, d],\ i < p$.

Definition 1.2 [33]**.** If $f \in B_{pq}(a,\,c)$ $(p, q \in \mathbb{N},\ p+q=m)$ then f admits Taylor's representation

$$f(x,y) = \sum_{i+j<m} \frac{(x-a)^i}{i!}\frac{(y-c)^j}{j!} f^{(i,j)}(a,c) + (R_m f)(x,y), \tag{1}$$

with

$$\begin{aligned}(R_m f)(x,y) = &\sum_{j<q} \frac{(y-c)^j}{j!}\int_a^b \frac{(x-s)_+^{m-j-1}}{(m-j-1)!} f^{(m-j,j)}(s,c)ds \\ &+ \sum_{i<p}\frac{(x-a)^i}{i!}\int_c^d \frac{(y-t)_+^{m-i-1}}{(m-i-1)!} f^{(i,m-i)}(a,t)dt \\ &+ \iint_D \frac{(x-s)_+^{p-1}}{(p-1)!}\frac{(y-t)_+^{q-1}}{(q-1)!} f^{(p,q)}(s,t)dsdt,\end{aligned}$$

where $z_+ = \begin{cases} z, & \text{when } z \ge 0, \\ 0, & \text{when } z < 0. \end{cases}$

Remark 1.1. The Taylor-type formula (1) holds for any domain Ω having the property that there exists a point $(a,\,c) \in \Omega$ such that the rectangle $[a, x] \times [c, y] \subseteq \Omega$, $(x,\,y) \in \Omega$.

Theorem 1.1 [33] (Peano's theorem)**.** *Let $f \in B_{pq}(a,\,c)$, $(p, q \in \mathbb{N}, p+q=m)$, $D = [a, b] \times [c, d]$, and the linear functional L given by*

$$\begin{aligned}L(f) = &\sum_{i+j<m} \iint_D f^{(i,j)}(x,y)d\mu_{i,j}(x,y) \\ &+ \sum_{j<q}\int_a^b f^{(m-j,j)}(x,c)d\mu_{m-j,j}(x) \\ &+ \sum_{i<p}\int_c^d f^{(i,m-i)}(a,y)d\mu_{i,m-i}(y)dy,\end{aligned} \tag{2}$$