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Isolation effects in a system of two mutually communicating identical patches

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ABSTRACT

Starting from the Fisher–Kolmogorov–Petrovskii–Piskunov equation (FKPP) we model the dynamic of a diffusive system with two mutually communicating identical patches and isolated of the remaining matrix. For this system we find the minimal size of each fragment in the explicit form and compare with the explicit results for similar problems found in the literature. From this comparison emerges an unexpected result that for a same set of the parameters, the isolated system studied in this work with size L, can be better or worst than the non isolated systems with the same size L, uniquely depending on the parameter a_0 (internal conditions of the patches). Due to the fact that this result is unexpected we propose an experimental verification.

1. Introduction

In the study of population dynamics, it is used many tools like metapopulations [9,21], diffusive systems [2,7], with one [20] and more [6] species interacting in many forms [1,5,17,19].

The problem, of a single species moving in a diffusive pattern is largely [4,8,12,15,16] modeled in literature by the equation of Fisher–Kolmogorov–Petrovskii–Piskunov (FKPP), that in one dimension is given by [3,10,11,18,20]:

$$\frac{\partial \Phi}{\partial t} = D\frac{\partial^2 \Phi}{\partial x^2} + a(x)\Phi - b\Phi^2, \tag{1}$$

where $\Phi = \Phi(x,t)$ is the population density, t is the time, x is the spatial variable, D is the diffusion coefficient, $a(x)$ is the growth rate and b is a saturation constant (related to the carrying capacity).

The function $a(x)$ is used to describe spatial heterogeneity, where we assume $a(x) > 0$ as a life region, a zone good for life (patch, island, fragment). If $a(x) < 0$, we assume as a death region, which is unfavorable for life. The profiles described in Figs. 1 and 2 represent examples of fragmented regions.

Using Ludwig arguments [14], reinforced in the literature [10,18], we consider the stead state of FKPP, Eq. (1) and neglected the nonlinear term $-\Phi^2$, to find the limit conditions between life region and death region. These considerations generate the equation:

$$D\frac{d^2 \Phi}{dx^2} + a(x)\Phi = 0. \tag{2}$$

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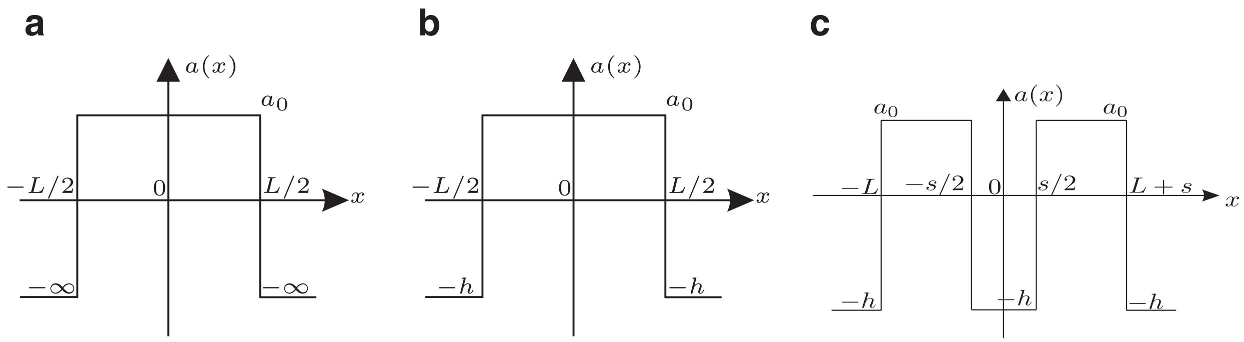


Fig. 1. Representation of fragmented regions: **a:** one patch isolated from the matrix; **b:** one patch non isolated from the matrix; **c:** two identical patches immersed in a matrix, separated by a region of length s. In these three cases, the internal conditions of the patches are a_0, their lengths are L and the life difficulty in the matrix is quantified by parameter h, except in the case of isolated systems, where life is impossible in the matrix.

Many profiles of heterogeneity can be interesting to population dynamics because they represent real systems, but if the function $a(x)$ assumes strange forms, the solution of Eq. (2) can be difficult and unfeasible to find. One simple form of $a(x)$ interesting to the study of population dynamics is the piecewise constant function. In this case, we assume homogeneous regions where $a(x) > 0$ like a patch and regions (homogeneous too) where $a(x) < 0$ like the matrix or the separation between two neighbor patches such as those in Fig. 1.

In the literature, it is possible to find profiles of $a(x)$ as piecewise constant function used to interpret biological growth systems. For example, there is the profile for one patch isolated of the matrix, Fig. 1a, which the minimal size patch was found by Skellam [20] and confirmed by Kenkre and Kuperman [11], satisfying:

$$L_{si} = \pi\sqrt{\frac{D}{a_0}}. \tag{3}$$

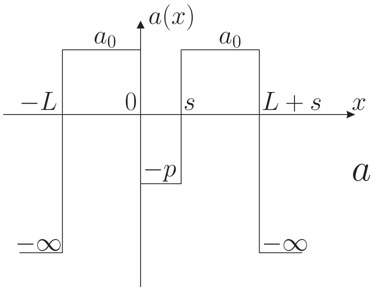
Another example of one patch profile, but non isolated from the matrix, Fig. 1b, was studied by Ludwig [14] who presented an expression for the minimum size of the fragment, in the form:

$$L_{sh} = 2\sqrt{\frac{D}{a_0}}\arctan\sqrt{\frac{h}{a_0}}. \tag{4}$$

There are studies for infinite numbers of patches [10,13], but one interesting case that has an explicit form for minimal island size is the case of two identical fragments immersed in the matrix, Fig. 1c, it was proposed by Kenkre and Kumar [10] who predicted Eq. (5):

$$L_{dh} = \sqrt{\frac{D}{a_0}}\left\{\arctan\sqrt{\frac{h}{a_0}} + \arctan\left[\sqrt{\frac{h}{a_0}}\tanh\left(\sqrt{\frac{h}{D}}\frac{s}{2}\right)\right]\right\}. \tag{5}$$

In this article, we propose two identical patches isolated from the matrix, but mutually communicating, which is the main propose of this work. This profile is represented in Fig. 2.



$$a(x) = \begin{cases} \text{region I:} & -\infty, \quad \text{if } -L < x \\ \text{region II:} & a_0, \quad \text{if } -L < x < 0 \\ \text{region III:} & -p, \quad \text{if } 0 < x < s \\ \text{region IV:} & a_0, \quad \text{if } s < x < L + s \\ \text{region V:} & -\infty, \quad \text{if } x < L + s \end{cases} \tag{6}$$

Fig. 2. Representation of an isolated system with two fragments mutually communicating. L is the size of the patches, a_0 the internal growth rate and p is the life difficulty level between the patches.