Applied Mathematics and Computation 315 (2017) 500–506

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A modified reproducing kernel method for solving Burgers' equation arising from diffusive waves in fluid dynamics

Ming-Jing Du[a,*], Yu-Lan Wang[b,*], Chao-Lu Temuer[b,c], Dan Tian[b]

[a] Institute of Computer Information Management, Inner Mongolia University of Finance and Economics, Hohhot 010070, China
[b] Department of Mathematics, Inner Mongolia University of Technology, Hohhot 010051, China
[c] Department of Mathematics, Shanghai Maritime University, Shanghai 201306, China

ARTICLE INFO

Keywords:
Reproducing kernel
Burgers' equation
Piecewise
Approximate solution

ABSTRACT

As we known, reproducing kernel method (RKM) has been presented for solving differential equations for initial and boundary value problems. However, the direct application of the RKM presented in the previous works cannot produce good numerical results for Burgers' equation. To solve this problem, this paper give a modified reproducing kernel method by piecewise technique. The exact solution is given by reproducing kernel functions in a series expansion form, the approximation solution is expressed by n-term summation of reproducing kernel functions. The three numerical experiments results show that the piecewise method is more easily implemented and effective. Some numerical results are also compared with the results obtained by other methods.

1. Introduction

We consider Burgers' equation

$$\frac{\partial u}{\partial t}+\alpha u\frac{\partial u}{\partial x}-\nu\frac{\partial^2 u}{\partial x^2}=0,\quad (x,t)\in(a,b)\times(0,T), \tag{1}$$

with the initial and boundary conditions

$$u(x,0)=\gamma(x),\quad x\in(a,b), \tag{2}$$

$$u(a,t)=g_0(t),\ u(b,t)=g_1(t),\quad t\in[0,T], \tag{3}$$

where $\nu > 0$ is a small parameter known as the kinematics viscosity and α is some positive constant, $\gamma(x)$, $g_0(t)$, $g_1(t)$ are known functions. Burgers' equation is the simplest nonlinear partial differential equation for diffusive waves in fluid dynamics. This model arises in many physical problems including one-dimensional turbulence, sound waves in a viscous medium, shock waves in a viscous medium, waves in fluid filled viscous elastic tubes, and magneto-hydrodynamic waves in a medium with finite electrical conductivity. Burgers' equation is similar to the one dimensional Navier–Stokes equation without the stress term. A great deal of effort has been expended in the last few years to compute efficiently the numerical solutions

* Corresponding authors.
E-mail addresses: 724297269@qq.com (M.-J. Du), wylnei@163.com (Y.-L. Wang).

http://dx.doi.org/10.1016/j.amc.2017.07.065
0096-3003/

Table 1
Comparison of the five methods solutions of Experiment 1 at $\nu = 0.1,\ t = 0.1 \times 10^{-4}$.

x	Exact solution	Present method	Old RKM [8]	ADM [9]	HBI [9]	Fourier [9]
0.1	0.30902	0.30902	0.30901	0.30901	0.30901	0.30900
0.2	0.58779	0.58779	0.58777	0.58778	0.58778	0.58776
0.3	0.80903	0.80902	0.80810	0.80901	0.80901	0.80899
0.4	0.95107	0.95107	0.95099	0.95106	0.95105	0.95104
0.5	1.00001	1.00001	0.99989	1.00001	0.99999	0.99999
0.6	0.95107	0.95107	0.95186	0.95108	0.95105	0.95106
0.7	0.80903	0.80903	0.80871	0.80904	0.80901	0.80902
0.8	0.58779	0.58780	0.58735	0.58781	0.58778	0.58779
0.9	0.30902	0.30902	0.30842	0.30903	0.30901	0.30902

of Burgers' equation for small and large values of the kinematic viscosity. Many researchers have solved Burgers'equation by various method, such as modified cubic B-splines collocation method [1], factorized diagonal Pade approximation [2], A novel numerical scheme [3], spectral collocation method [4], Sinc Differential Quadrature Method [5], Polynomial based differential quadrature method [6], Quartic B-splines Differential Quadrature Method [7] etc.

The authors in [14–16] used the RKM to solve Burgers' equation. Xie et al. [14] present a numerical method for solving one-dimensional Burgers' equation by the Hopf–Cole transformation and a reproducing kernel function. Li et al. [15,16] used the RKM for solving one-dimensional homogeneous variable coefficient Burgers' equation by an iterative technique, and obtained the accurate numerical results. In [17], The authors used the RKM for solving some nonlinear differential-difference equations. In [18], The authors used the generalized differential quadrature method for solving Burgers'equations. In [12,13], the author used reproducing kernel for solving a class of time-fractional telegraph equation by piecewise technique, but the initial value conditions are all zero. This paper present the piecewise reproducing kernel method for solving Burgers' equation with initial and boundary conditions based on [12,13], and obtain more effective numerical solution by piecewise technique.

The paper is organized as follows: in Section 2, we list the solution of Burgers' equation, convergence analysis and error estimate. In Section 3, three numerical experiments show that the piecewise method is more efficient. Conclusion is given in Section 4 that briefly summarizes the numerical outcomes.

In order to solve (1)–(3) by RKM, the initial and boundary value conditions are need to be homogenized. Let $y = (x - a)/(b - a)$, the problem can be transformed to the following problem

$$\begin{cases} \frac{\partial u}{\partial t} + \alpha u \frac{\partial u}{\partial y} - \nu \frac{\partial^2 u}{\partial y^2} = 0, & (y,t) \in (0,1) \times (0,T), \\ u(y,0) = u_0(y), \quad u(0,t) = g_0(t), & u(1,t) = g_1(t), \end{cases} \tag{4}$$

where $u_0(y) = \gamma(a + (b - a)y)$.

Put $U(y,t) = u(y,t) - w(y,t) - u_0(y) + w_0(y)$, where $w(y,t) = g_0(t)(1 - y) + g_1(t)y$, $w_0(y) = w(y,0)$, so we can get

$$\begin{cases} \frac{\partial U}{\partial t} + \frac{\alpha}{(b-a)} U \frac{\partial U}{\partial y} - \frac{\nu}{(b-a)^2} \frac{\partial^2 U}{\partial y^2} = f\left(y,t,U,\frac{\partial U}{\partial y}\right), & (y,t) \in (0,1) \times (0,T), \\ U(y,0) = 0, \quad U(0,t) = 0, \quad U(1,t) = 0, \end{cases} \tag{5}$$

where

$$f\left(y,t,U,\frac{\partial U}{\partial y}\right) = -\frac{\partial w}{\partial t} - \frac{\alpha U}{b-a}\frac{\partial (w + u_0 - w_0)}{\partial y} - \frac{\alpha}{b-a}\frac{\partial (U + w + u_0 - w_0)}{\partial y}(w + u_0 - w_0) + \frac{\nu}{(b-a)^2}\frac{\partial^2 (w + u_0 - w_0)}{\partial y^2}.$$

For simplicity, we replace U with u and y with x in (5), and $T = 1$.

2. Solution of Burgers' equation

The reproducing kernel space $W_2^2[0,1]$, $W_2^3[0,1]$, $H(D) = W_2^3[0,1] \otimes W_2^2[0,1]$ and $H_1(D) = W_2^1[0,1] \otimes W_2^1[0,1]$ has defined in [8], and we can get the exact solution

$$u(x,t) = \sum_{j=1}^{\infty} f\left(x_j, t_j, u(x_j,t_j), \frac{\partial u(x_j,t_j)}{\partial x}\right)\zeta_j(x,t). \tag{6}$$

Because Burgers' equation is nonlinear, the approximate solution is (see [8])

$$u_n(x,t) = \sum_{j=1}^{\infty} f\left(x_j, t_j, u_{n-1}(x_j,t_j), \frac{\partial u_{n-1}(x_j,t_j)}{\partial x}\right)\zeta_j(x,t), \ n = 2, \ldots. \tag{7}$$