Applied Mathematics and Computation 315 (2017) 591–602

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Theoretical study on continuous polynomial wavelet bases through wavelet series collocation method for nonlinear Lane–Emden type equations

S.C. Shiralashetti*, S. Kumbinarasaiah

Department of Mathematics, Karnatak University, Dharwad 580003, India

ARTICLE INFO

MSC:
42C40
34A12
65N35

Keywords:
Legendre wavelet
Hermite wavelet
Laguerre wavelet
Lane–Emden type equations
Collocation method
Isomorphism

ABSTRACT

In this article, a new method is generated to solve nonlinear Lane–Emden type equations using Legendre, Hermite and Laguerre wavelets. We are interested to note that these wavelets will give same solutions with good accuracy. Theorems on convergence analysis are stated and proved on the spaces, which are created by Legendre, Hermite and Laguerre wavelets bases and justified these spaces are equivalent to polynomial linear space generated by general polynomial basis. The main idea for obtaining numerical solutions depends on converting the differential equation with initial and boundary conditions into a system of linear or nonlinear algebraic equations with unknown coefficients. A very high level of accuracy reflects the reliability of this scheme for such problems.

1. Introduction

Wavelets theory is a newly emerging area in mathematical research field. It has been applied in engineering disciplines; such as signal analysis for wave form representation and segmentations, time-frequency analysis, harmonic analysis etc. Wavelets permit the accurate representation of a variety of functions and operators. Moreover, wavelets establish a connection with fast numerical algorithms [1–7].

The application of Chebyshev wavelets for solving differential and integral equations is thoroughly considered by many authors [8,9]. Also, Legendre wavelets are used for solving some differential and integral equations [10–12]. In recent years, the studies of singular initial and boundary value problems for second order ordinary differential equations (ODEs) have attracted the attention of many mathematicians and physicists. One of the equations describing this type is the Lane–Emden type equation. Since Lane–Emden type equations have significant applications in many scientific fields, various forms have been investigated in many research works. Among them, many attentions have been carried on the generalized Lane–Emden type equations,

$$y'' + \frac{\alpha}{x}y' + f(x,y) = g(x), \quad 0 < x \leq 1, \quad \alpha \geq 0, \tag{1}$$

subjected to initial and boundary conditions,

$$\begin{cases} y(0) = \alpha_1, & y'(0) = \alpha_2 \\ y(a) = \alpha_3, & y'(b) = \alpha_4 \end{cases} \tag{2}$$

* Corresponding author.
E-mail addresses: shiralashettisc@gmail.com, shiralashettisc@yahoo.com (S.C. Shiralashetti).

http://dx.doi.org/10.1016/j.amc.2017.07.071

where α_1,α_2,α_3,α_4,a, b are constants, $f(x, y)$ is continuous real valued function, and $g(x) \in C[0, 1]$ and analytic solution of the Eq. (1) is always possible [13] in the neighbourhood of the singular point $x = 0$ for the above initial condition. Eq. (1) was named after the astrophysicists Jonathan H. Lane and Robert Emden (1870), as it was first studied by them [14]. Eq. (1) was used to model several phenomena in mathematical physics and astrophysics such as the theory of stellar structure, the thermal behaviour of a spherical cloud of gas, isothermal gas sphere, and theory of thermionic currents.

Singular IVPs and BVPs of Lane-Emden type has been investigated by a large number of authors. A new algorithm for solving differential equations of Lane-Emden type [15], A new analytic algorithm of Lane-Emden type equations [16], The numerical method for solving differential equations of Lane-Emden type by Pade approximation [17], A Jacobi rational pseudo-spectral method for Lane–Emden initial value problems arising in astrophysics on a semi-infinite interval [18], sinc-collocation method [19], an implicit series solution [20], a Jacobi-Gauss collocation method [21], Second kind Chebyshev operational matrix algorithm [22], New spectral second kind Chebyshev wavelets algorithm [23], New Wavelets collocation method [24], New Ultraspherical wavelets spectral solutions [25], New spectral solutions [26] and New ultraspherical wavelets collocation method [27]. Recently, some other approximate solutions of Lane-Emden type equations are obtained using perturbation techniques [28], optimal homotopy method [29], generalized Jacobi-Galerkin method [5,30].

Spectral methods play notable roles in extracting the solution for different kinds of differential equations. Tau, collocation and Galerkin methods are three most widely used spectral methods. Collocation method is one of the famous techniques for solving differential equations; in particular, this method provides highly accurate solutions to nonlinear differential equations [31]. According to our survey, no one has been solved nonlinear Lane–Emden equations using Legendre, Hermite and Laguerre wavelets series collocation method.

The paper is organized as follows; Section 2 is devoted to the Preliminaries on wavelets. Function approximation and convergence analysis is presented in Section 3. Wavelet based numerical method of solution is given in Section 4. Numerical experiment is presented in Section 5. Section 6 deals with the concluding remarks of the paper.

2. Wavelets

Wavelets [30] constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter $\boldsymbol{a}$ and the translation parameter $\boldsymbol{b}$ varies continuously, family of continuous wavelets are,

$$\psi_{a,b}(x) = |a|^{\frac{-1}{2}}\ \psi\left(\frac{x-b}{a}\right),\ \ a, b \in R,\ \ a \neq 0.$$

If we restrict the parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ to discrete values as $a = a_0^{-k},\ \ b = nb_0a_0^{-k}, a_0 > 1,\ \ b_0 > 0$, family of discrete wavelets are,

$$\psi_{k,n}(x) = |a_0|^{\frac{1}{2}}\ \psi\left(a_0^k x - nb_0\right).$$

Where $\psi_{k,n}$ forms a wavelet basis for $L^2(R)$. In particular, when $a_0 = 2$ and $b_0 = 1$, then $\psi_{k,n}(x)$ forms an orthonormal basis.

2.1. Laguerre wavelet

The Laguerre wavelets [32] $\psi_{k,n}(x) = \psi(k, n, m, x)$ involve four arguments, $n = 1, 2, 3,...,2^{k-1}$, k is assumed any positive integer, m is the degree of the Laguerre polynomials and it is the normalized time. They are defined on the interval [0, 1) as,

$$\psi_{n,m}(x) = \begin{cases} 2^{\frac{k}{2}} \bar{L}_m\left(2^k x - 2n + 1\right), & \frac{n-1}{2^{k-1}} \leq x < \frac{n}{2^{k-1}} \\ 0 & otherwise \end{cases}$$
$$Where\, L_m(x) = \frac{L_m}{m!}$$

$m = 0, 1, 2...M-1$. Here $L_m(x)$ are the Laguerre polynomials of degree m with respect to the weight function $W(x) = 1$ on the interval $[0, \infty)$ and satisfy the following recursive formula, $L_0(x) = 1,\ \ L_1(x) = 1 - x$,

$$L_{m+2}(x) = \frac{(2m+3-x)L_{m+1}(x) - (m+1)L_m(x)}{m+2},\quad m = 0, 1, 2$$

2.2. Hermite wavelet

The Hemite wavelets [33] $\psi_{k,n}(x) = \psi(k, n, m, x)$ involve four arguments, $n = 1, 2, 3, ..., 2^{(k-1)}$, k is assumed any positive integer, m is the degree of the Hemite polynomials and it is the normalized time. They are defined on the interval $[0,\infty)$ as,

$$\psi_{n,m}(x) = \begin{cases} \frac{2^{\frac{k+1}{2}}}{\pi^{\frac{1}{2}}} H_m\left(2^k x - 2n + 1\right), & \frac{n-1}{2^{k-1}} \leq x < \frac{n}{2^{k-1}} \\ 0 & otherwise \end{cases}$$