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Numerical investigation of electrohydrodynamic instability in a vertical porous layer

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ABSTRACT

The electrohydrodynamic instability of a vertical dielectric fluid saturated Brinkman porous layer whose vertical walls are maintained at different temperatures is considered. An external AC electric field is applied across the vertical porous layer to induce an unstably stratified electrical body force. The stability eigenvalue equation is solved numerically using the Chebyshev collocation method. The presence of inertia is found to instill instability on the system and the value of modified Darcy–Prandtl number Pr_D at which the transition from stationary to travelling-wave mode takes place is independent of the AC electric field but increases considerably with an increase in the value of Darcy number *Da*. The presence of AC electric field promotes instability but its effect is found to be only marginal. Although the flow is stabilizing against stationary disturbances with increasing *Da*, its effect is noted to be dual in nature if the instability is via travelling-wave mode. The streamlines and isotherms for various values of physical parameters at their critical state are presented and analyzed. Besides, energy norm at the critical state is also computed and found that the disturbance kinetic energy due to surface drag, viscous force and dielectrophoretic force have no significant effect on the stability of fluid flow.

1. Introduction

Natural convection in a layer of fluid saturated porous medium has constituted a pole of attraction to researchers because of its applications in several fields such as oil recovery, soil mechanics, rheology, metal casting, ceramic engineering, solid-matrix compact heat exchangers, spread of contaminants in the environment and in various processes in the chemical and materials industry, and the technologies of paper, textiles, insulating materials. Available works on this topic are concerned mostly with the study of natural convection in horizontal porous layers heated from below. The state of the art has been summarized in the books by Nield and Bejan [1] and Straughan [2,3].

The stability of natural convection in a vertical porous layer has been investigated by many authors. Gill [4] was the first to investigate the stability properties of natural convection in a vertical layer of Darcy porous medium and his findings established a rigorous ground for the use of insulating porous materials in buildings, instead of air gaps. Kwok and Chen [5] discussed different qualitative behaviors of the Darcy flow model used by Gill [4] considering the full Darcy–Brinkman model with advective inertia and also performed experiments. Qin and Kaloni [6] used energy methods to give

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Nomenclature

a	vertical wave number
c	wave speed
c_r	phase velocity
c_i	growth rate
$D=d/dx$	differential operator
$Da=\mu_e K/\mu h^2$	Darcy number
$\vec{E}$	root-mean-square value of the electric field
E_b, E_d, E_D, E_e, E_s	disturbance kinetic energies
E_0	root-mean-square value of the electric field at $x=0$
$\vec{f}_e$	force of electrical origin
$\vec{g}$	acceleration due to gravity
h	half- width of the porous layer
K	permeability
p	pressure
$P = p - 0.5\rho(\partial\varepsilon/\partial\rho)(\vec{E}\cdot\vec{E})$	modified pressure
$Pr_D = \nu h^2\varphi_p^2/K\kappa$	modified Darcy–Prandtl number
$\vec{q} = (u, v, w)$	velocity vector
$R_D = \alpha g\beta h^2 K/\nu\kappa$	Darcy–Rayleigh number
$R_{eaD} = \gamma^2\varepsilon_0 E_0^2\beta^2 h^2 K/\mu\kappa$	AC electric Darcy–Rayleigh number
t	time
T	temperature
T_c, T_d	disturbance thermal energies
T_1	temperature of the left vertical boundary
T_2	temperature of the right vertical boundary
V	root-mean-square value of the electric potential
V_1	electric potential of the left vertical boundary
V_2	electric potential of the right vertical boundary
W_b	basic velocity
(x, y, z)	Cartesian coordinates

Greek symbols

α	thermal expansion coefficient
$\beta=(T_2-T_1)/h$	temperature gradient
γ	thermal expansion coefficient of dielectric constant
ε	dielectric constant
ε_0	reference dielectric constant at T_0
κ	thermal diffusivity
χ	ratio of heat capacities
μ	dynamic viscosity
μ_e	effective fluid viscosity
$\nu(=\mu/\rho_0)$	kinematic viscosity
$\psi(x, z, t)$	stream function
Ψ	amplitude of vertical component of perturbed velocity
ϕ	amplitude of perturbed electric potential
φ_p	porosity of the porous medium
ρ	fluid density
ρ_e	free charge density
ρ_0	reference density at T_0
σ	electrical conductivity of the fluid
θ	amplitude of perturbed temperature

sufficient conditions for the stability of convection in a vertical porous slab. Most of the developments are covered in the book by de Lemos [7]. Barletta [8] reconsidered Gill's problem and showed that the change of velocity boundary conditions from impermeable to permeable causes instability while Barletta [9] studied two-dimensional stationary mixed convection in a vertical porous layer. Rees [10] and Scott and Straughan [11] presented a new perspective on Gill's problem by taking into account of the local thermal nonequilibrium effect, while Shankar and Shivakumara [12,13] extended Gill's and Rees's problems for an Oldroyd-B type of viscoelastic fluid, respectively. Recently, the effect of inertia on the stability of