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Low Mach number preconditioning techniques for Roe-type and HLLC-type methods for a two-phase compressible flow model

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A B S T R A C T

We describe two-phase flows by a six-equation single-velocity two-phase compressible flow model with stiff mechanical relaxation. In particular, we are interested in the simulation of liquid–gas mixtures such as cavitating flows. For the numerical approximation of the homogeneous hyperbolic portion of the model equations we have previously developed two-dimensional wave propagation finite volume schemes that use Roe-type and HLLC-type Riemann solvers. These schemes are very suited to simulate the dynamics of transonic and supersonic flows. However, these methods suffer from the well known difficulties of loss of accuracy and efficiency encountered by classical upwind finite volume discretizations at low Mach number regimes. This issue is particularly critical for liquid–gas flows, where the Mach number may range from very low to very high values, due to the large and rapid variation of the acoustic impedance. In this work we focus on the problem of loss of accuracy of standard schemes related to the spatial discretization of the convective terms of the model equations. To address this difficulty, we consider the class of preconditioning strategies that correct at low Mach number the numerical dissipation tensor. First we extend the approach of the preconditioned Roe–Turkel scheme of Guillard–Viozat for the Euler equations [Computers & Fluids, 28, 1999] to our Roe-type method for the two-phase flow model, by defining a suitable Turkel-type preconditioning matrix. A similar low Mach number correction is then devised for the HLLC-type method, thanks to a novel reformulation of the HLLC solver. We present numerical results for two-dimensional liquid–gas channel flow tests that show the effectiveness of the proposed preconditioning techniques. In particular, we observe that the order of pressure fluctuations generated at low Mach number regimes by the preconditioned methods agrees with the theoretical results inferred for the continuous relaxed two-phase flow model by an asymptotic analysis.

1. Introduction

We describe two-phase flows by a six-equation single-velocity two-phase compressible flow model with stiff pressure relaxation [1,2]. In particular, we are interested in the simulation of liquid–gas mixtures such as cavitating flows, which are found in many systems of different areas of engineering, such as naval and aerospace technologies and the nuclear industry.

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In the modeling of these flows it is important to take into account the compressibility of all the phases, gas as well as liquid, to correctly describe acoustic perturbations and the wave dynamics. This has motivated our study of a two-phase flow model that treats each phase as a compressible fluid. This model, which we first considered in [1,3], is a variant of the six-equation two-phase flow model with stiff pressure relaxation of Saurel–Petitpas–Berry [2], and it belongs to a class of compressible multiphase flow models stemming from the original work of Baer–Nunziato [4], see e.g. the models in [5,6].

We numerically approximate the two-phase system by a fractional step algorithm, which alternates between the solution of the homogeneous hyperbolic portion of the equations through wave propagation Godunov-type finite volume schemes, and the solution of a system of ordinary differential equations that takes into account the stiff pressure relaxation source terms. For the solution of the homogeneous two-phase system we have proposed in [1] schemes that use Roe-type and HLLC-type Riemann solvers. These numerical methods prove to be very efficient to simulate wave propagation phenomena and shocks in transonic and supersonic flows. Unfortunately, these schemes suffer from the well known difficulties encountered by upwind finite volume discretizations for compressible flows at low Mach number: dramatic loss of accuracy, failure of convergence to the correct solution, very high computational cost when standard time-explicit schemes are used, since the CFL stability condition for these schemes demands a time step of the order of the Mach number. These issues are particularly critical for liquid–gas flows, since here the Mach number may range from very low values in the weakly compressible liquid medium to very large values in the gas and liquid–gas mixture zones. Due to the large and rapid variation of the acoustic impedance in these flows, it is important to be able to accurately describe a wide range of Mach number regimes.

There exists very abundant literature on the topic of low Mach number flows, including theoretical studies on the incompressible limit of the compressible flow equations, analyses of the failure of some classical compressible flow solvers, and formulations of numerical methods suited for low Mach number regimes by a variety of approaches [7–16,16–25]. Note that most of the studies on this topic are devoted to the case of single-phase flows (Euler or Navier–Stokes equations), while theoretical and numerical work on multiphase flow models is still limited in the literature, cf. e.g. [26–30]. This paper will focus solely on the problem of loss of accuracy at low Mach number, which is related to the spatial discretization of the convective terms of the considered model equations. In particular the problem was examined in depth for Godunov-type schemes for the Euler equations by Guillard, Murrone and Viozat in a series of papers [12,13,31]. In [12] the authors explain via an asymptotic analysis that the loss of accuracy is linked to the generation in the discrete solutions of pressure fluctuations of the wrong order of magnitude in the Mach number, with respect to the behavior of the continuous flow model. In [12] the proof is rigorously given for the Roe's scheme. A powerful remedy to cure the accuracy problem of upwind finite volume schemes is preconditioning of the numerical dissipation term. Preconditioning techniques were originally introduced [9, 32] to accelerate convergence to a steady state and first used for low-speed steady state computations [10,33,34]. Preconditioning consisted of a matrix multiplying the time derivative term with the effect of reducing the disparity of the eigenvalues corresponding to the convective and acoustic modes, thus removing the stiffness of the equations. These techniques were not suited for time-dependent problems. It was then observed that preconditioning improves not only convergence but also accuracy of the steady state solution for low Mach number problems [34]. This evidence led to the development of preconditioning strategies that do not alter the time derivative term as in the first approaches but act only on the upwind dissipation term in order to cure the accuracy problem while retaining consistency in time [12,13,35], thus allowing the simulation of unsteady flows. In this paper we consider this class of preconditioned methods. Note that these methods remain very inefficient in time [36] when explicit time integrations are used, and suitable time discretizations must be employed. However here we will not address this issue, which is nonetheless an important point of our planned work. Most of the low Mach number strategies that alter the dissipation term to improve accuracy have been conceived for approximations of the single-phase Euler equations [12,13,20,35]. Some methods have also been developed for simple homogeneous mixture models [37–40], which describe a two-phase medium as a single fluid governed by a set of equations formally monophasic. Work on preconditioning techniques for genuine multiphase compressible flow models of Baer–Nunziato type like the model that we consider is still scarce in the literature. The work of Murrone and Guillard [28] was the first to study theoretically at the continuous level the low Mach number behavior of the five-equation single-velocity single-pressure two-phase flow model of Kapila et al. [5], and to introduce a preconditioned method for this two-phase system based on an acoustic Riemann solver. More recently some preconditioning strategies have been proposed for similar two-phase flow models [29,30].

Following the work of Guillard, Viozat and Murrone [12,28,35], the aim of our work has been to devise preconditioning techniques for the Roe-type and HLLC-type schemes that we have previously developed for our two-phase flow model, and to provide further insight on the accuracy problem for the two-phase case by examining the low Mach number behavior of continuous and discrete solutions of the equations. The main result of our studies has been the extension of the Roe–Turkel scheme of Guillard–Viozat [12] for the Euler equations to our original Roe-type scheme for the two-phase flow model, via the definition of a suitable Turkel-type preconditioner acting on the numerical viscosity matrix. Then, thanks to a novel reformulation of the HLLC Riemann solver that reveals the mathematical analogy of its wave structure with the one of the Roe solver, we were able to extrapolate the approach of the Roe–Turkel method of [12] to HLLC-type discretizations and obtain an HLLC–Turkel method for the two-phase flow model. A detailed study of this new HLLC–Turkel technique for the classical single-phase Euler equations will be presented in a separate work. Two-dimensional computations of low Mach number channel flow problems show the effectiveness of the proposed preconditioning techniques. In particular, we observe that the order of pressure fluctuations generated at low Mach number by the Roe–Turkel and HLLC–Turkel methods for the two-phase flow model agrees with the theoretical results inferred for the continuous equations.