Applied Mathematics and Computation 310 (2017) 134–138

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Logit selection promotes cooperation in voluntary public goods game

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A R T I C L E I N F O

Keywords:
Game theory
Nonlinear dynamics
Complex networks
Cooperation

A B S T R A C T

In this paper, we present a simple yet effective model to promote cooperation in selfish population, namely, a spatial evolutionary public goods game model that includes three kinds of players: cooperator, defector and loner. In spatial settings, the players locate on a regular lattice, and each player randomly selects one strategy, then all the player acquire their payoffs with their four nearest neighbors, after that the focal player chooses a neighbor based on the logit selection model and updates his/her strategy in accordance with a random sequential simulation procedure. The Monte Carlo simulation results demonstrate that the ruthless invasion of defectors can be efficiently prevented by the loners, especially when enhanced factor r is low. Further interesting is the fact that the introduction of a logit selection model, making the fittest neighbors more likely to act as sources of adopted strategies, effectively promotes the evolution of cooperation even if the loner is absence.

1. Introduction

In behavioral sciences and social economic, the evolution of cooperation among unrelated individuals represents one of the most stunning phenomena [1–6]. The prisoner's dilemma (PD) and snowdrift game (SD) has long been established as paradigms to explain cooperative behavior through pairwise interactions [7–9]. While the PD and SD have attracted attention from biologists and social scientists, most studies in experimental economics focused on the closely related yet more general public goods game for group interactions, which can be treated as a multi-person pairwise interactions. When the number of players is equal to 2, it will degenerate into the PD game [10–13]. In a typical public goods game, there are five players, each of which has the fixed wealth. Then the players have the opportunity to invest their money into a common pool or not. Here the player investing his/her money is a cooperator, otherwise the player is defector. After that, the total amount in the pool is enhanced by a fixed factor r and equally divided among all participants, irrespective of their contributions. Interesting, defection represents the dominating strategy leading to the "rational" equilibrium. Obviously, the public goods game is abundant in animal and human societies, such as predator inspection behavior, alarm calls, and group defense, as well as health insurance, public transportation, or environmental issues [14,15], to name only a few.

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http://dx.doi.org/10.1016/j.amc.2017.04.027
0096-3003/

Aim to solving the above tragedy and investigating the reason why cooperators can survive, several mechanisms have been proposed. The pioneering work of Nowak gave a new prospect for the behavioral evolution [2], namely, the cooperation can survive via forming cluster in networks to resist the exploration of defection. Soon, some other social mechanisms such as reward and punishment [16,17], reputation [18,19], personnel diversity [20,21] have been put forward. Besides, the interaction topology has been intensively studied as a paradigm for cooperative behavior in spatial public goods game, such as scale free networks [22,23], interdependent networks [24–26] and weighted network [27–29], which have proved valuable for the tragedy to some extent.

In the present work, the loner who refuses to participate in the public goods game is taken into account. In addition, given the instinct of people learning from the better in the strategy update step, the player chooses a neighbor is not random, but according to the famous logit selection model [30,31]. In the remainder of this article we will first describe the evolutionary public goods game model. Subsequently, we will present the main results, and finally we will summarize our conclusions.

2. Model

In this article, we study the evolutionary public goods game on a square lattice with three kinds of players, namely, cooperator, defector and loner. The public goods game is played in a population with N players. The public goods game model is a well-known multi-person social dilemma [31]. Here we assume that the contribution of cooperators ($s_x = C$) is equal to 1 while defectors ($s_x = D$) contribute nothing, but the loners ($s_x = L$) refuse to participate and rather rely on some small but fixed income σ [31]. The sum of all contributions in the group is multiplied by the enhanced factor r, and the resulting public goods are distributed among all the group members. Correspondingly, the payoff of player x is:

$$P_x = \begin{cases} \sigma, & if\ s_x = L \\ \frac{r \sum_{s_y=C} I_y}{N} - 1, & if\ s_x = C \\ \frac{r \sum_{s_y=C} I_y}{N}, & if\ s_x = D \end{cases} \tag{1}$$

In order to accelerating the simulations but without causing major modifications, we assume that the score of player x is determined by a single public goods game that involves the focal player and its four nearest neighbors. Namely, a player located on a square lattice and interact with their four nearest neighbors lies in the north, east, south and west [21,31]. The game is iterated forward in accordance with a random sequential simulation procedure including three steps. First, a random selected player x acquires its payoff p_x by playing the public goods game with all its neighbors. Secondly, one of the neighbor y of player x is chosen according to the logit model [32], namely the probability is:

$$\Pi_y = \frac{\exp\left(w p_y\right)}{\sum_z \exp\left(w p_z\right)}. \tag{2}$$

Here, w is a tunable parameter. Evidently, for $w = 0$, the logit model turns into the situation where player y is chosen uniformly at random from all the neighbors of player x. However, if $w > 0$, Eq. (2) introduces a preference toward those neighbors of player x that have higher payoff p_y. Conversely, for $w < 0$, players with a lower payoff are more likely to be selected as potential strategy donors. Lastly, player x adopts the strategy s_y from the selected neighbor y with the probability [33]:

$$W(s_y \to s_x) = \frac{1}{1 + \exp\left[(p_x - p_y + \tau)/K\right]}, \tag{3}$$

where $\tau > 0$ denotes the cost of strategy change and K introduces some noise to allow for irrational. For the sake of simplicity, we set $\tau = 0.1$ in this simulation.

Results of computer simulations presented below were obtained on populations comprising 130×130 individuals, whereby the fraction of cooperators was determined within 10,000 full iteration steps after sufficiently long transients were discarded. Moreover, since the preferential selection of neighbors may introduce additional disturbances, final results were averaged over up to 10 independent runs for each set of parameter values in order to assure suitable accuracy.

3. Results

Firstly, we inspect the fraction of strategies in dependence on the enhanced factor r for different value of $w$in Fig. 1, where the black curve is corresponding to the fraction of cooperators, the red curve is corresponding to the fraction of defectors, the green curve is corresponding to the fraction of loner. It is clear that positive values of w promote the evolution of cooperation, on the other hand, negative values of w impede it. Besides, the critical enhanced factor r_L, which making the extinction of loners is also decreased by the value of tunable parameter w, and after the critical enhanced factor r_L, loners go extinct because they no longer provide a valuable alternative. Also the break point of defection r_D is also decrease with the increasing of tunable parameter w. It is interesting to see that the promotive effect on the survivability of cooperators becomes more potent monotonously with the increasing of w. So it is can be concluded that as the increasing of tunable parameter w, the promotion effect of cooperation increased.