Applied Mathematics and Computation 310 (2017) 194–203

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A conservative spectral collocation method for the nonlinear Schrödinger equation in two dimensions

Rongpei Zhang[a,*], Jiang Zhu[b], Xijun Yu[c], Mingjun Li[d], Abimael F.D. Loula[b]

[a] School of Mathematics and Systematic Sciences, Shenyang Normal University, Shenyang 110034, PR China
[b] Laboratório Nacional de Computação Científica, MCTI, Avenida Getúlio Vargas 333, 25651-075 Petrópolis, RJ, Brazil
[c] Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing 100088, PR China
[d] School of Mathematics and Computation Science, Xiangtan University, Hunan, 411105, China

ARTICLE INFO

Keywords:
Spectral collocation method
Nonlinear Schrödinger equation
Compact integration factor method
Conservation

ABSTRACT

In this study, we present a conservative Fourier spectral collocation (FSC) method to solve the two-dimensional nonlinear Schrödinger (NLS) equation. We prove that the proposed method preserves the mass and energy conservation laws in semi-discrete formulations. Using the spectral differentiation matrices, the NLS equation is reduced to a system of nonlinear ordinary differential equations (ODEs). The compact implicit integration factor (cIIF) method is later developed for the nonlinear ODEs. In this approach, the storage and CPU cost are significantly reduced such that the use of cIIF method becomes attractive for two-dimensional NLS equation. Numerical results are presented to demonstrate the conservation, accuracy, and efficiency of the method.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction

The nonlinear Schrödinger (NLS) equation plays an important role in many fields of physics, such as plasma physics, quantum physics and nonlinear optics [4,7]. In this paper, we consider the following cubic NLS equation

$$i\frac{\partial u}{\partial t} + \Delta u + \beta |u|^2 u = 0, \tag{1}$$

on two-dimensional domain $\Omega = [a, b] \times [c, d]$ with periodic boundary conditions and initial condition

$$u(x, y, 0) = u_0(x, y). \tag{2}$$

Here $i = \sqrt{-1}$ is the complex unit, $u = u(x, y, t)$ is a complex-valued function, Δ is the Laplace operator, and β is a given real constant. It is easy to show that the periodic-initial value problem (1) and (2) admits mass conservation law

$$Q(t) = \int_\Omega |u(x, y, t)|^2 dxdy \equiv \int_\Omega |u_0(x, y)|^2 dxdy = Q(0) \tag{3}$$

and the energy conservation law

$$E(t) = \int_\Omega \left(|\nabla u(x, y, t)|^2 - \frac{\beta}{2} |u(x, y, t)|^4 \right) dxdy \equiv \int_\Omega \left(|\nabla u_0|^2 - \frac{\beta}{2} |u_0|^4 \right) dxdy = E(0) \tag{4}$$

for $t > 0$.

* Corresponding author.
E-mail addresses: rongpeizhang@163.com, rongpeizhang@gmail.com (R. Zhang), jiang@lncc.br (J. Zhu), yuxj@iapcm.ac.cn (X. Yu), alimingjun@163.com (M. Li), aloc@lncc.br (A.F.D. Loula).
http://dx.doi.org/10.1016/j.amc.2017.04.035
0096-3003/© 2017 Elsevier Inc. All rights reserved.

Extensive numerical methods have been studied for the NLS equation in the literature. These methods include the spectral (pseudospectral) method [3,23], finite difference method (FDM) [6,8,9,25,26,28], finite element method (FEM) [1,10,12], discontinuous Galerkin (DG) method [15,29,30]. Recently, Li et al. [16] studied a sixth-order alternating direction implicit method for two-dimensional Schrödinger equation. Shi et al. [21] studied the superconvergence analysis of conforming finite element method for NLS equation. Kong et al. [14] proposed a compact and efficient conservative scheme for coupled NLS equations. Taleei and Dehghan [22] studied the pseudo-spectral domain decomposition method in space discretization and time-splitting method in time discretization for NLS equation. Katsaounis and Mitsotakisc [13] used an implicit-explicit type Crank–Nicolson finite element scheme to study the phenomenon of soliton reflection. For more numerical methods the reader may consult the Ref. [2].

It has been well known that spectral methods can provide a very useful tool for the solution NLS equation because of their spectral accuracy, when the geometry of the problem is smooth and regular [5]. The Fourier pseudo-spectral method is one of the spectral methods which is applied for various nonlinear Schrödinger equations by many authors such as Muruganandam and Adhikari [18], and Bao et al. [3]. Recently, Fourier [17] and Chebyshev [11,22] spectral collocation method were proposed for solving NLS equation. These collocation methods have spectral accuracy and can be proved to keep conservation. However, these methods are limited in one-dimensional case. In this paper, we apply Fourier spectral collocation method for solving two-dimensional NLS equation. After the space discretization by the spectral differentiation matrices, the two-dimensional NLS equation is discretized into a system of nonlinear ordinary differential equations (ODEs) in matrix formulation. We will prove that the proposed method preserve the discrete mass and energy conservation laws.

In the time integration, the alternating direction implicit (ADI) schemes are usually applied in FDM discretization which consists of a number of tridiagonal matrix equations [9,28]. However, the spectral collocation method is not appropriate for the ADI method because the spectral differentiation matrices are full. Here we consider the compact implicit integration factor (cIIF) method for time discretization [19,20,31]. In the space discretization, the diffusion terms are approximated by the spectral differentiation matrix. Then the cIIF method applies matrix exponential operations sequentially in x- and y-direction. As the results, we can calculate and store the exponential in small sizes. Another novel property of the methods is that the exact evaluation of the diffusion terms is decoupled from the implicit treatment of the nonlinear terms. We only solve a local nonlinear system at each spatial grid point.

The rest of the paper is organized as follows: in Section 2 we present the Fourier spectral collocation method combined with compact integration factor method to solve NLS equation in 2D. We also prove the mass and energy conservation laws in semi-discrete formulation. Numerical experiments are reported in Section 3. Finally, we summarize our conclusion in Section 4.

2. Numerical methods

2.1. Fourier spectral collocation method

In the process of spectral collocation method, the essential part is the generation of the spectral differentiation matrix. We first give the Fourier spectral differentiation matrix on the interval $[0, 2\pi]$. Other intervals can be easily handled by a scale factor.

Let N be positive even integer. The spacing of the grid is $h = 2\pi/N$ and the collocation points are $\{x_j = jh,\ \ j = 1, 2, \ldots, N\}$. We assume that $f(x)$ is a function on $[0, 2\pi]$. Then $f(x)$ can be interpolated by a sum of periodic sinc functions [24]

$$f_N(x) = \sum_{j=1}^{N} f(x_j) S_N(x - x_j), \tag{5}$$

where $S_N(x) = \frac{\sin\frac{\pi x}{h}}{\frac{2\pi}{h}\tan\frac{x}{2}}$. The function $S_N(x)$ is viewed as periodic sinc function which interpolates the discrete delta function

$$\delta_j = \begin{cases} 1, & j = 0(\bmod N), \\ 0, & j \neq 0(\bmod N). \end{cases}$$

The derivatives of the interpolant $f_N(x)$ are then estimated at collocation points by differentiating (5)

$$f'_N(x) = \sum_{j=1}^{N} f(x_j) S'_N(x - x_j). \tag{6}$$

Define the vector of function and derivatives as $F = [f(x_1), f(x_2), \ldots, f(x_N)]^T$ and $F^{(1)} = [f'_N(x_1), f'_N(x_2), \ldots, f'_N(x_N)]^T$, respectively. We can get the matrix formulation of (6) on collocation points

$$F^{(1)} = D_N F, \tag{7}$$

Download English Version:

https://daneshyari.com/en/article/5775691

Download Persian Version:

https://daneshyari.com/article/5775691

Daneshyari.com