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Distributed adaptive fixed-time consensus tracking for second-order multi-agent systems using modified terminal sliding mode

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ABSTRACT

This paper investigates the fixed-time consensus tracking problems for second-order multi-agent systems with unknown external disturbances. Firstly, a fixed-time terminal sliding mode (FTTSM) is proposed, which can avoid the singularity problem. Then, two continuous distributed consensus tracking control laws with adaptive updating laws are designed respectively, in which the upper bounds of external disturbances are not required. It is proved that the two control laws can both guarantee the consensus tracking errors converge into the desired regions including the origin in fixed time. A simulation example is given to demonstrate the effectiveness of proposed methods.

1. Instruction

Recently, distributed cooperative control of multi-agent systems (MASs) has been paid to much attention due to its potential applications in spacecraft formation flying, multi-unmanned air vehicles formation, multi-sensor fusion, and so forth [1–4]. Two view points in them are mainly considered, one is the distributed regulation problem (leaderless consensus), that is how to design a protocol such that all states of agents converge to an unprescribed common value [5,6]; the other is the distributed tracking problem (leader–follower consensus), that is how to design a protocol such that all states of agents converge to a desired common value [7,8]. Note that the leader–follower consensus can not only save the energy, but also enhance the communication and orientation of the flock compared with leaderless consensus, thus many researchers focus on the consensus tracking problems for first-order and second-order MASs [9–19]. However, the protocols proposed in there can only guarantee the closed-loop system is asymptotically stable.

For consensus tracking problems, one significant requirement is the fast convergence rate. Compared with the asymptotic control approaches, the finite-time control approaches can not only provide fast convergence rate but also provide higher tracking precision and better disturbance-rejection ability [20–27]. Therefore, many finite-time control laws are proposed for various MASs in the past few years [28–34]. For example, the authors of [28–32] designed the finite-time control laws for first-order and second-order MASs using the homogeneous method [31] and the Lyapunov-based method [28,32], respectively. Unfortunately, the settling time can not be estimated for the homogeneous method, and the settling time can be estimated dependent on the initial conditions of systems for the Lyapunov-based method. In practical applications, we desire that the settling time are estimated independent on the initial conditions of systems since the controller can be

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designed such that some control performances can be satisfied in a fixed time and regardless to initial conditions. Moreover, the needing of initial conditions may prevent us estimating the settling time under distributed control [38]. Therefore, many scholars studied finite-time control laws design using the fixed-time stable concept [35–37], where the settling time was estimated without requiring the knowledge of initial conditions. [38–41] designed the fixed-time control laws for first-order and second-order MASs, respectively, but the external disturbances were not considered in there, especially for disturbances with unknown upper bounds [43–46]. Inspired by the above discussions, in this paper, we will further investigate the adaptive finite-time consensus tracking problems for second-order MASs with upper bounds unknown external disturbances using fixed-time terminal sliding mode.

Compared with most of the existing finite-time consensus tracking control methods, the main contributions of this paper are summarized as follows:

1) The communications among agents are described by a directed graph, which will bring more challenges than the case that the communications among agents are described by a undirected graph.
2) A new fixed-time terminal sliding mode (FTTSM) is proposed for second-order MASs, which can avoid the singularity problem.
3) Two continuous adaptive consensus tracking control laws are given such that the consensus tracking errors converge into a small neighborhood of the origan in fixed time, in which the upper bounds of external disturbances are not required and the chattering is avoided.

The rest of the paper is organized as follows. In Section 2, the systems dynamics and mathematical preliminaries are presented. The main results are presented in Section 3. Simulation results and conclusions are given in Section 4 and 5, respectively.

Throughout this paper, $\mathbb{R}^{n\times m}$ denotes the $n \times m$-dimensional Euclidean spaces; $\otimes$ denotes the Kronecker product; I denotes the identity matrix with appropriate dimensions; the superscripts T and -1 stand for matrix transposition and matrix inverse, respectively; $\text{sig}^r(\cdot) = \text{sgn}(\cdot)|\cdot|^r$ and $\text{sgn}(\cdot)$ is the sign function.

2. System dynamics and mathematical preliminaries

2.1. Graph theory

Consider a network with n agents and assume that the communications among them are described by a weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, in which $\mathcal{V} = \{1, 2, \dots, n\}$ is the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, respectively. $(i, j) \in \mathcal{E}$ means that there is an edge from node i to node j, and $N_i = \{j|(v_j, v_i) \in \mathcal{E}\}$ denotes the set of neighbors of node i. The weighted adjacency matrix of $\mathcal{G}$ is defined as $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} > 0$ for $(j, i) \in \mathcal{E}$, $a_{ij} = 0$ for $(j, i) \notin \mathcal{E}$ and $a_{ii} = 0$ for $\forall i$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = D - A$, in which $D = \text{diag}\{d_1, d_2, \dots, d_n\}$ is the in-degree matrix with $d_i = \sum_{j=1}^{n} a_{ij}$. If there exists a sequence of successive edges in the form of $\{(i, k), (k, l), \dots, (m, j)\}$, we say that there exists a direct path from node i to node j. Moreover, if there is a node (called the root node) such that a directed path is existed from this node to every other node in the graph, we call the digraph has a spanning tree .

Consider an extension graph $\bar{G} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ associated with the system consisting of n followers and one leader. Denote the leader adjacency matrix as $B = \text{diag}\{b_1, \dots, b_n\}$, in which b_i is a strictly positive constant if there is an edge from the node of leader to node i, otherwise, $b_i = 0$.

2.2. System description

This paper considers the networked MAS with n followers and one leader, and the communications among them are described by a digraph $\bar{G}$. The dynamic of the ith follower is modeled as

$$\begin{aligned} \dot{q}_i &= v_i \\ \dot{v}_i &= u_i + w_i, i \in \mathcal{V} \end{aligned} \tag{1}$$

where $q_i \in \mathbb{R}^p, v_i \in \mathbb{R}^p, u_i \in \mathbb{R}^p, w_i \in \mathbb{R}^p$ are the position vector, velocity vector, control input vector and external disturbance vector, respectively.

Assumption 1. There exists an unknown positive constant w_i^* such that w_i satisfies $\|w_i\| \le w_i^*$.

Denote $q_d \in \mathbb{R}^p$ as the state vector of leader and it's first time derivative $\dot{q}_d$ and second time derivative $\ddot{q}_d$ are all assumed to be smooth, bounded and known functions. The following assumption is further required.

Assumption 2. $\bar{\mathcal{G}}$ contains a spanning tree with the root node being the leader node.

2.3. Some lemmas

Lemma 1 [10]**.** *All the eigenvalues of matrix $H = L + B$ have positive real parts if and only if Assumption 2 holds.*