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Synchronization of delayed neural networks with hybrid coupling via partial mixed pinning impulsive control

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ABSTRACT

This paper investigates the synchronization problem of a class of general hybrid coupling delayed neural networks with internal delay as well as coupling delay. A general hybrid coupling term involving current-state coupling, discrete-delay coupling and distributed-delay coupling is considered. The partial mixed pinning impulsive control is proposed for achieving synchronization, which is combined with the advantage of pinning impulsive control and two impulsive effects simultaneously (i.e. synchronization and desynchronization). In order to handle the difficulties of multi-time delays, some generalized differential inequalities about time-varying delays are established. By using Lyapunov functional method and applying a mixed pinning impulsive control scheme, some sufficient conditions are derived to guarantee global synchronization of the neural networks. Moreover, our results can cover and extend the previous related works. Finally, numerical examples are also given to illustrate the efficiency of our methods and the theoretical results.

1. Introduction

Over the past decades, coupled delayed neural networks, as a special kind of complex networks, have been studied extensively because they have many important applications in various areas such as signal processing, moving image processing, optimization, speed detection of moving subjects, secure communication and so on, see [1–8] and references therein. Synchronization, as a typical collective behavior in the dynamics of neural networks, naturally has been paid increasing attentions. In biological systems, such as synchronous fireflies, swarming of fishes, flocking of birds, synchronization has received more and more observations. What's more, it has been shown that the specific brain function or the critical physiological state such as epilepsy and Parkinsons disease in neuroscience [9,10] are often linked to the presence or absence of synchrony in the brain. Therefore, it has become one of the hottest topics in the studies of synchronization behaviors in an array of coupled neural networks.

In general, control techniques have great influence on the realization of network synchronization. Hitherto, various control schemes have been used to study the synchronization of complex networks, such as adaptive control [11], H_∞ control [12,13], impulsive control [14–17], intermittent control [18,19] and pinning control [20–24]. Among these control methods, pinning control is undoubtedly a powerful technique because it is effective and relatively easily to be realized by controlling a small fraction of the nodes instead of all the nodes in whole network. Impulsive control is one of the important discontinuous control schemes which is practical in simulating the abrupt changes at certain instants. Compared with continuous

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control schemes, impulsive control is energy-saving since it allows systems to possess discontinuous inputs. By considering the merits of pinning control and impulsive control, pinning impulsive control was proposed to investigate the synchronization of coupled complex networks which can be used as a simple and effective strategy to stabilize or synchronize the underlying dynamical networks [25–28]. For example, a simple generic criterion of impulsive pinning synchronization for coupled oscillator network is derived in [25]. In [26], the problem of pinning a complex dynamical network to the solution of an uncoupled system was investigated by applying impulsive control to a small fraction of network nodes. In [27], the authors investigated the synchronization problem for a class of complex dynamical networks with directed or undirected but a strongly connected topology using pinning impulsive scheme. Recently, He et al. [28] proposed a novel node selection strategy to investigated the synchronization of coupled neural networks with distributed-delay coupling via pinning impulsive control. Actually, there exists two kinds of impulsive effects for synchronization in the pinning impulsive control process: (i) synchronizing impulses which can enhance the synchronization of dynamical networks; and (ii) desynchronizing impulses which suppress the synchronization of dynamical networks. These two types of impulsive control existing together are called mix pinning impulsive control [29]. However, only one type synchronizing impulses was taken into account in the most of previous relative works about impulsive pinning control. In this paper, both synchronizing impulses and desynchronizing impulses simultaneously will be considered in the studying of synchronization.

It should be pointed out that time delay is inevitable in practical dynamical networks due to the finite speeds of transmission and spreading as well as traffic congestion. Generally speaking, there exist two types of time delay in dynamical networks: one is internal delay occurring inside the dynamical node, and the other is coupling delay caused by the exchange of information between dynamical nodes. In order to be more practical, the coupling schemes involving time delays should contain discrete-delay coupling and distributed-delay in modeling transmittal delays. Studies have been made on synchronization of coupled neural networks with both current-state coupling and distributed-delay coupling in [28]. A novel node selection strategy was given according to the norm of error signal between the follower and the leader in [27] and [29] to make complex networks get to synchronization, and the mathematical model of such dynamic networks was delay-free. Pinning impulsive synchronization of time-delayed dynamic network was investigated in [30,31]. As far as we know, most of existing results concerned synchronization of coupled networks with only internal delay or coupling delay or part of them, such as in [32] the internal delay and in [28,33] the coupling delay was considered respectively. In this paper, a class of more general neural networks with hybrid time-delay will be considered, which contains internal delay and two kinds of coupling delay: discrete-delay and distributed-delay, and they are supposed to be time-varying.

Motivated by the discussions above, this paper aims to investigate synchronization of a class of delayed neural networks with hybrid coupling via partial mixed impulsive control. The coupling terms in the networks model are composed of current-state coupling, discrete-delay coupling and distributed-delay coupling. Synchronizing effects and desynchronizing effects are considered in the pinning impulsive control. Based on mix pinning impulsive control technique and Lyapunov theory, some sufficient criterions are derived to guarantee global synchronization of the neural networks. In addition, some corollaries are presented which show our main results are less conservative and more general than previous ones. Finally, numerical examples are provided to illustrate the effectiveness of the theoretical results. The contribution of this paper can be summarized as follows: (1) a generalized comparison principle is given by strict theory analysis, in which multi-time-delays are taken into account; (2) the inner delay, discrete-delay and distributed-delay are considered in our model, and all of them are time-varying; (3) both synchronizing and desynchronizing impulsive effects are considered, by which the influence of impulse on dynamical network systems will be comprehensive analyzed.

The rest of this paper is organized as follows. In Section 2, the general coupled neural networks model with hybrid time-varying delays and some necessary definitions, lemmas, and notations are given. In Section 3, we establish some synchronization criteria for the proposed general coupled neural networks model. Furthermore, some corollaries are presented to show extensions of our results. Then, two numerical examples illustrating the validity of of the theoretical results are presented in Section 4. Finally, we draw some conclusions in Section 5.

2. Model description and preliminaries

The linearly coupled neural networks model with hybrid time-varying delays could be described as follows:

$$\begin{aligned}\dot{x}_i(t) &= -Dx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau_1(t))) + c_1\sum_{j=1}^{N} b_{ij}^{(1)}\Gamma_1 x_j(t)\\ &\quad + c_2\sum_{j=1}^{N} b_{ij}^{(2)}\Gamma_2 x_j(t-\tau_2(t)) + c_3\sum_{j=1}^{N} b_{ij}^{(3)}\Gamma_3\int_{t-\tau_3(t)}^{t} x_j(s)ds + J,\end{aligned} \tag{1}$$

where $i = 1, 2, \dots, N$ and $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$ denotes the state of the ith neural network. $D = diag\{d_1, d_2, \dots, d_n\} > 0$ denotes the rate with which the ith cell resets its potential to the resting state when it is isolated from other cells and inputs, and $f(x_i(t)) = (f_1(x_i(t)), f_2(x_i(t), \dots, f_n(x_i(t))))^T$ is the activation function at time t. The positive constants c_1, c_2 and c_3 are the coupling strengths. Γ_1, Γ_2 and $\Gamma_3 \in \mathbb{R}^{n\times n}$ are the inner connecting matrices. $B^{(1)} = (b_{ij}^{(1)})_{N\times N}, B^{(2)} = (b_{ij}^{(2)})_{N\times N}$ and $B^{(3)} = (b_{ij}^{(3)})_{N\times N}$ represent the coupling configuration of the network defined to satisfy diffusive coupling: $b_{ij}^{(k)} \ge 0 (i \ne j)$ and $b_{ii}^{(k)} = -\sum_{j=1, i\ne j}^{N} b_{ij}^{(k)}, k = 1, 2, 3$. The time-varying delays $\tau_1(t)$, $\tau_2(t)$, and $\tau_3(t)$