CrossMark

Spatiotemporal dynamics and spatial pattern in a diffusive intraguild predation model with delay effect ☆

Renji Han, Binxiang Dai*

School of Mathematics and Statistics, Cental South University, Changsha 410083, PR China

A R T I C L E   I N F O

A B S T R A C T

Based on biological meaning, a kind of diffusive intraguild predation (IGP: resource, IG prey and IG predator) model with delay effect is investigated in this paper. The model has Holling-Type I functional response between resource-IG prey and resource-IG predator; Holling-Type II functional response between IG prey and IG predator. We first give sufficient conditions on the stability of possible nonnegative constant steady-state solutions for the proposed model, which give us a complete picture of the global dynamics. Then we investigate Hopf bifurcation near the unique positive constant steady-state solution by taking delay as bifurcation parameter and derive the Hopf bifurcation threshold. It is shown that the delay can induce three types of bistability (node-node bistability, node-cycle bistability and cycle-cycle bistability), periodic oscillations and irregular oscillations triggering spatiotemporal chaos in the diffusive IGP model. Numerical simulations are performed to illustrate our theoretical results and suggest that delay can even trigger the emergence of self-organised spatiotemporal patterns, which evolve from spiral patterns to irregular spatial patterns via spatiotemporal Hopf bifurcation. In addition, the impact of diffusion on the model's dynamics under certain time delay are also explored.

1. Introduction

The interaction among populations is one of the basic interspecies relations in biology and ecology (see e.g. [1]). Intraguild predation (IGP)is a widespread ecological phenomenon in which two consumers species that share a resource also engaged in a predator-prey interaction. IGP is a common occurrence in natural communities [2–9]. Since the pioneering work of Polis and others [2,4,5], many papers on IGP models or population interaction models have been published to illustrate the importance of this interaction ([10,11]).

In [5], the first general mathematical models describing IGP was exploited by Holt and Polis. Their model takes the following form

$$\begin{cases} \dot{R}(t) = R(\varphi(R) - \rho_1(R,N,P)N - \rho_2(R,N,P)P), \\ \dot{N}(t) = N(e_1\rho_1(R,N,P)R - \rho_3(R,N,P)P - m_1), \\ \dot{P}(t) = P(e_2\rho_2(R,N,P)R + e_3\rho_3(R,N,P)N - m_2), \end{cases} \tag{1.1}$$

where $R(t)$, $N(t)$, $P(t)$ represent the densities of the basal resource, the IG prey and IG predator, respectively. The quantities $\rho_2(R, N, P)R$ and $\rho_3(R, N, P)N$ are functional responses of the IG predator to the resource and IG prey, respectively; $\rho_1(R, N, P)R$ is the functional response of the IG prey to the basal resource; and m_1 and m_2 are density-independent morality rates. The parameters e_1 and e_2 convert resource consumption into reproduction for the IG prey and IG predator, respectively; the parameter e_3 scales the benefit enjoyed by the IG predator from its consumption of IG prey; $R\varphi(R)$ is recruitment of the basal resource.

Functional response describes how the consumption rate of individual consumers varies with respect to resource density and is often used to model predator-prey interactions. Some authors investigated some questions for IGP models with different types of functional responses. Velazquez et al. [12] and Hsu et al. [13] investigated the case with a linear functional response for the three predation terms. Abrams and Fung [14] investigated prey persistence and mainly provided numerical simulation for a kind of IGP models with Holling type-II functional responses. Verdy and Amarasekare [15] and Freeze et al. [16] investigated Holling type-II and ratio-dependent functional responses, respectively. Kang and Wedekin [17] considered Holling-III functional response for predation on the IG prey by the IG predator in two kinds of models, which one is called IGP model with specialist predator and the other is called IGP model with generalist predator.

The models mentioned above only assume that the predation population can instantaneously convert the consumption into its growth. In fact, the reproduction of predator after consuming the prey is not instantaneous, but is mediated by some reaction-time lag required for gestation. Time delay is an important factor in ecology because complicate dynamical behavior can often be induced by it. It has been proved that such time delay can destabilize a stable equilibrium and induce oscillations and even chaos in some population models, such as predator-prey models and food web models (see [18–25] and the references therein). It should be pointed out that IGP models involve both predator-prey interactions and competition. Hence, it is of great interest to study the dynamics of IGP models when delay is presented. Very recently, Shu et al. [19] investigated the complex dynamics of the following IGP model with delay:

$$\begin{cases} \dot{R}(t) = rR(t)\left(1 - \dfrac{R(t)}{K}\right) - c_1 R(t)N(t) - c_2 R(t)P(t), \\ \dot{N}(t) = e_1 c_1 R(t-\tau)N(t-\tau) - c_3 N(t)P(t) - m_1 N(t), \\ \dot{P}(t) = e_2 c_2 R(t)P(t) + e_3 c_3 N(t)P(t) - m_2 P(t), \end{cases} \tag{1.2}$$

where r is the growth rate of R in the absence of N and P, K is the carrying capacity of resource. c_1 is the predation rate of IG prey for resource, c_2 is the predation rate of IG predator for resource, c_3 is the maximum population of IG prey killed by IG predator. τ is the gestation delay. The other parameters have the same meaning as model (1.1).

In reality, the species is spatially heterogeneous and individuals will tend to migrate towards regions of lower population density to add the possibility of survival, and hence the species are distributed over space and interact each other within their spatial domain. The importance of spatial models has been acknowledged by the biologists, chemical experimenters and applied mathematicians for a long time and have been one of the dominant topic in both ecology and mathematical ecology because of its universal existence and importance ([1,26–31,40,43]). For models with two species or three species, the stability and the Hopf bifurcations of constant steady state solutions for reaction-diffusion equations with delay effect have been considered extensively in one dimensional domain or two dimensional domains (see [33–36] and the references therein). Tian et al. [37] and Li and Wang [38] investigated Hopf bifurcation in reaction-diffusive model with delay effect in higher dimensional bounded domains. It should be emphasized that the Hopf bifurcation analysis in higher dimensional domains is more difficult than one dimensional domain. Hence, it is of great interest to consider the effect of delay on diffusive intraguild predation model in higher dimensional domains. To the best of our knowledge, there are no results on spatiotemporal dynamics and pattern in a delayed diffusive IGP model with Holling-II functional response and higher dimensional domains.

Now, we begin to formulate our model. Suppose $\Omega \subset \mathbb{R}^n (n \geq 1)$ is a bounded domain with smooth boundary $\partial\Omega$. Let $R(t, x)$, $N(t, x)$, $P(t, x)$ represent the densities of the basal resource, the IG prey and IG predator at time t and location x, respectively. The basal resource is assumed to grow logistically with intrinsic growth rate r and the carrying capacity K. We assume the basal resource is consumed by the IG prey and IG predator at rates $c_1 R(t, x)N(t, x)$ and $c_2 R(t, x)P(t, x)$ at time t and location x, respectively. We also assume that when the IG prey population is low, the killed IG prey is simply proportional to both the IG prey $N(t, x)$ and the IG predator $P(t, x)$, but that the IG predator saturates when the IG prey is sufficiently high. Moreover, the reproduction of IG predator after preying upon IG prey is not instantaneous, but is mediated by some constant time lag τ for gestation [19], which is modeled as $\frac{e_3 c_3 P(t-\tau, x)N(t-\tau, x)}{1+aN(t-\tau, x)}$. Similarly, the reproductions of IG predator and IG prey after preying up the basal resource are $e_1 c_1 R(t-\tau, x)N(t-\tau, x)$ and $e_2 c_2 R(t-\tau, x)P(t-\tau, x)$, respectively. The meanings of $e_i (i = 1, 2, 3)$ and m_i are the same as those in model (1.1). In addition, the time-lag parameter is introduced in a manner analogous to that in the model (1.2). Simultaneously, we assume that the populations are in an isolate patch and neglect the impact of migration, including immigration and emigration, and the populations cannot across